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Stability and convergence in strongly monotone dynamical systems.

The author observes that there is an abundance of significant monotone dynamical systems on strongly ordered subsets of topological spaces. These examples include solution flows of ordinary and partial differential equations in various function spaces, in particular in spaces of almost periodic functions. As an instance, the solution flows for the quasilinear parabolic systems of H. Amann [SIAM Rev. **18** (1976), no. 4, 620–709; MR0415432; erratum; MR0467410] are strongly monotone. In addition to showing the relevance of the present work to these particular examples, the paper reveals further properties of these flows.

The paper is not limited to flows on Banach spaces but such a state space is convenient to describe the results. Let $X$ be a nonempty open set in a real Banach space, $V$. Assume that $V$ has a partial ordering defined by a closed convex cone, $V_+ \subset V$, with $V_+ \cap (-V_+) = \{0\}$. $V_+$ is assumed to have a nonempty interior. Such a space is said to be strongly ordered. A flow $\varphi$ on $X$ is called monotone if $\varphi_t y \geq \varphi_t x$ whenever $y > x$ and $t \geq 0$, and strongly monotone if $\varphi_t y - \varphi_t x \in \text{Int} V_+$ whenever $y > x$ and $t > 0$.

A theme of the paper is to show that strongly monotone flows are numerous, significant and nonchaotic. Usually, the orbits with compact closure are stable and attracted to the set of stationary points. Let $\varphi$ be a strongly monotone flow on an open set, $X$, of a strongly ordered separable Banach space, $V$, and assume that every orbit has compact closure in $X$. Let $\omega(x)$ denote the omega limit set of $x \in X$. Let $E \subset X$ be the set of stationary points and $Q$ the set of quasiconvergent points (i.e., $x \in Q$ if, and only if, $\omega(x) \subset E$). According to a major theorem there are very few nonquasiconvergent orbits in $X$ in the sense of topology, measure or cardinality. The author proves that $Q$ is residual, $\mu(X \setminus Q) = 0$ for every Gaussian measure $\mu$ on $V$ and $J \setminus Q$ is countable for every simply ordered arc $J \subset X$.

These results use no conditions of differentiability nor any genericity hypotheses. With further assumptions even stronger consequences are secured.

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