Let $\varphi: \mathbb{R} \to \mathbb{R}$ be of class $C^{2+\gamma}$, $\gamma > 0$, and satisfy $\varphi(x + 1) = \varphi(x) + 1$, $\varphi'(x) \geq \text{const} > 0$. Suppose the rotation number $\rho$ of the induced circle mapping $T_\varphi$ is irrational with continued fraction expansion $\rho = [k_1, \ldots, k_n, \ldots]$ satisfying $k_n \leq \text{const} n^{\nu}$, $\nu > 0$. The theorem of the title is: $T_\varphi$ is $C^1$-conjugate to rotation of the circle through angle $\rho$. The point of the result is smoothness; topological conjugacy was established by Denjoy in 1932. Write the equation for the density $\pi(x)$ of an absolutely continuous measure, assuming existence, as $\pi(T_\varphi(x))/\pi(x) = 1/\varphi'(x)$. Herman’s result is equivalent to the existence of a continuous strictly positive solution of this equation. The authors give a proof of this second result “based on the thermodynamic representation of dynamical systems and the study of ergodic properties of the corresponding random variables”. The authors, together with E. B. Vul’, have used such formalism previously in the study of Feigenbaum’s interval mappings [Uspekhi Mat. Nauk 39 (1984), no. 3(237) 3–37; MR0747790].

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