The system
\[ \frac{dx}{dt} = f(x, y), \quad \frac{dy}{dt} = g(x, y), \]
where \( f \) and \( g \) are of class \( C^n \) and period 1, defines a flow on the torus. Assuming an integral invariant \( U(x, y) \) (and tacitly assuming \( f^2 + g^2 > 0 \)) it is shown that this system can be transformed into one of the form \( \frac{dx}{dt} = F(x, y), \quad \frac{dy}{dt} = \gamma F(x, y) \). The constant \( \gamma \) is the ratio of the integral of \( Uf \) to that of \( Ug \) over the torus. In case \( F \) is analytic, the possibility of a further reduction to the form \( \frac{dx}{dt} = A, \quad \frac{dy}{dt} = B, \quad B/A = \gamma \), by an analytic transformation of coordinates is shown to depend on the arithmetic nature of \( \gamma \). It is sufficient that there exist positive constants \( K \) and \( h \) such that \( |m - n\gamma| > Kh^n \) for all integers \( m \) and \( n \). The latter result, as well as the former in the case of analytic flows, is due to Kolmogoroff [Dokl. Akad. Nauk SSSR (N.S.) 93 (1953), 763–766; MR0062892]. In the present paper, the hypothesis \( f^2 + g^2 > 0 \) is not stated, but must be assumed. Otherwise an example of a singular, ergodic, analytic flow on the torus given by the reviewer [Proc. Amer. Math. Soc. 4 (1953), 982–987; MR0060812] would contradict the first theorem. In this connection the correction [J. Math. Soc. Japan 4 (1952), 338; MR0053403] to a paper by Saitô cited by the author should also be noted.

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