A mapping \( T(z) = f(z, \mu) \) of \( \mathbb{C} \) into \( \mathbb{C} \) is considered, where \( f \) is a holomorphic function of \( z \) and \( \mu \) is a parameter. \( T^k \) stands for the \( k \)th iterate of \( T \). Analogously to period-doubling bifurcations, period-tripling bifurcations are studied via solutions of the functional equation (1) \( T^3(\varphi(z)) = \alpha^{-1} \varphi(\varphi(\varphi(\alpha z))), \varphi(0) = 1 \). By means of a perturbation method it is shown that (1) admits a fixed point \( \varphi_0(z), \alpha = \alpha_3,0 \), with only one eigenvalue \( \delta_3 \) of modulus larger than unity. It is concluded that period-tripling bifurcations are topologically stable and that the sequence of bifurcation values \( \mu_k \) admits the "universal" asymptotic estimate (2) \( \mu_\infty - \mu_k \approx C\delta_3^{-k}, C = \text{const} \). Since the convergence of the perturbation method is not examined, the conclusions are confirmed by a numerical determination of \( \varphi_0(z), \alpha_3,0 \) and \( \delta_3 \).

Numerical computations are also used to study the generalisation of (1) to period-quadrupling and period-quintupling bifurcations, and estimates of \( \alpha_4,0, \delta_4 \) and of \( \alpha_5,0, \delta_5 \) are obtained. It is conjectured that the same bifurcation pattern exists for period-\( n \)-tupling, \( n > 5 \).

It is claimed that at least the period-tripling bifurcation pattern is extendable to \( \mathbb{R}^2 \to \mathbb{R}^2 \) mappings not equivalent to mappings \( \mathbb{C} \to \mathbb{C} \). No supporting evidence is presented in favour of this claim.


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