This paper continues earlier researches on measure preserving transformations by von Neumann [Ann. of Math. (2) 33, 587–642 (1932)]. Two measure spaces, that is, spaces on certain sets of which measures are defined, are called point isomorphic if one can be transformed into the other in a 1–1 measure preserving way (neglecting a set of measure 0 in each). Two measure spaces are called set isomorphic if there is a 1–1 measure preserving transformation of the measurable sets of one into those of the other (sets of measure 0 disregarded throughout) which takes sums into sums and complements into complements. Throughout the following, “conditions of type $F$” will mean conditions on the fields of measurable sets of the measure spaces involved. (1) Necessary and sufficient conditions of type $F$ are found that a measure space be point isomorphic to the unit interval (with Lebesgue measure). (2) Under hypotheses of type $F$, necessary and sufficient conditions are found that every set automorphism of a measure space on itself be generated by a point transformation. (3) Under hypotheses of type $F$, it is shown that an ergodic measure preserving transformation $T$, with pure point spectrum, of a measure space into itself is point isomorphic to a rotation $x \rightarrow ax$ on a compact separable Abelian group (on which measure is Haar measure). This result makes possible simple proofs of properties of measure preserving transformations. Thus it is shown that $T$ is necessarily isomorphic to its own inverse. (4) For the group rotations of (3), metric transitivity is equivalent to regional transitivity. (5) If $T$ is an ergodic measure preserving transformation on a metric measure space $X$, with the usual relations between measure and metric, and if $T$ is isometric, or more generally, if the family $\{T^n\}$ is equicontinuous, then $T$ has a pure point spectrum; in fact, a multiplication can be defined on $X$ so that $X$ becomes a compact separable Abelian group, and $T$ becomes a rotation.

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