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Veech, William A. [Veech, William Austin]
★Projective Swiss cheeses and uniquely ergodic interval exchange transformations.
Ergodic theory and dynamical systems, I (College Park, Md., 1979–80), pp. 113–193,

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Veech, William A. [Veech, William Austin]
Gauss measures for transformations on the space of interval exchange maps.

Let \( n \geq 2 \) be integral. Let \( \pi \) be a permutation of the symbols \( \{1, \cdots, n\} \). We say that \( \pi \) is irreducible if \( \pi(\{1, \cdots, k\}) = \{1, \cdots, k\}, k \geq 1 \), implies \( k = n \). (This is not the usual definition of irreducibility.) Let \( \alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n) \) be a probability vector \((\alpha_k \geq 0, \sum_{k=1}^{n} \alpha_k = 1)\). The interval exchange transformation \( T(\alpha, \pi) \) is the map from the interval \([0, 1)\) to itself defined by cutting up \([0, 1)\) into (right open, left closed) intervals of lengths \( \alpha_1, \cdots, \alpha_n \) and permuting these intervals by \( \pi \). The reviewer has shown [Math. Z. 141 (1975), 25–31; MR0357739] that if \( \pi \) is irreducible and \( \alpha \) is irrational (i.e., \( \sum_{k=1}^{n} m_k \alpha_k = m_0, m_k \) integral, implies \( m_0 = m_1 = \cdots = m_n \)), then Kronecker’s theorem is valid for \( T(\alpha, \pi) \), i.e., the \( T(\alpha, \pi) \)-orbit of any point of \([0, 1)\) is dense in \([0, 1)\), and he conjectured [Israel J. Math. 26 (1977), no. 2, 188–196; MR0435353] that for any given irreducible \( \pi \), \( T(\alpha, \pi) \) satisfies the Weyl theorem, i.e., the \( T(\alpha, \pi) \)-orbit of any point in \([0, 1)\) is uniformly distributed in \([0, 1)\), for almost all probability vectors \( \alpha \).

In the first paper the author verifies this conjecture for \( n = 4 \), and in the second paper he shows that the conjecture is true for general \( n \). To do this, he defines a group action on the set of interval exchange transformations, using the inducing construction of ergodic theory, and shows that this group action possesses a unique invariant absolutely continuous measure, which naturally decomposes into ergodic components, one for each Rauzy class of irreducible permutations. Explicit formulae for the density of this measure (for different \( n \)) are given. These papers form an important and substantial contribution to ergodic theory.

{For the entire collection in which the first paper appears see MR0633759.}

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