The author considers a dynamical system defined on a 2-dimensional torus \( T^2 \) by the system of differential equations

\[
\frac{dx}{dt} = A(x,y), \quad \frac{dy}{dt} = B(x,y),
\]

and possessing an invariant integral \( I(g) = \int U(x,y) \, dx \, dy \), where \( A, B \) and \( U \) are univalued, analytic periodic functions of \( x \) and \( y \) with period \( 2\pi \). Here \( x \) and \( y \) are real coordinates mod \( 2\pi \), \( A^2 + B^2 > 0 \), \( U > 0 \) on the whole of \( T^2 \). It is then known [Nemyckii and Stepanov, Qualitative theory of differential equations, 2nd. ed., Gostehizdat, Moscow-Leningrad, 1949; for a review of the 1st ed. see MR0029483] that there exists an analytic transformation of coordinates which transforms the system (1) into the system

\[
\frac{dx}{dt} = \frac{1}{F(x,y)}, \quad \frac{dy}{dt} = \frac{\gamma}{F(x,y)}
\]

with an integral invariant \( I(g) = \int F(x,y) \, dx \, dy \), where \( \gamma \) is a constant.

The following theorem is asserted. Theorem 1. If there exist constants \( c > 0 \) and \( h > 0 \) such that for all positive integers \( m \) and \( n \)

\[
|m - n\gamma| \geq ch^n,
\]

then there exists an analytic transformation of coordinates which transforms the system (2) into the system

\[
\frac{du}{dt} = \lambda_1, \quad \frac{dv}{dt} = \lambda_2,
\]

where \( \lambda_1, \lambda_2 \) are constants and \( \lambda_2 = \gamma \lambda_1 \) and with the integral invariant \( I(g) = K \int F \, du \, dv \). Condition (i) is fulfilled for every \( \gamma \) except for a set of Lebesgue measure zero (\( c \) and \( h \) depend on \( \gamma \)). It follows that system (1) has a pure point spectrum with analytic proper functions.

For those irrational numbers which do satisfy (i) the author states: Theorem 2. Each of the following conditions is possible for a suitable choice of \( \gamma \) and \( F(x,y) \): The system (2) can be transformed into (3) by (I) an infinitely differentiable but not analytic transformation, (II) a \( k \)-differentiable but not \((k+1)\)-differentiable transformation, (III) an everywhere-discontinuous transformation; and (IV) the system (2) cannot be transformed into (3) at all. In (I), (II) and (III) the original system (1) has a pure point spectrum but the proper functions are respectively not analytic, not \((k+1)\) differentiable and everywhere discontinuous. The conjecture is made that in (IV) the spectrum is necessarily continuous but only a considerably weaker result is proved. In all statements related to Theorem 2 the notions of analyticity, differentiability, etc. are interpreted modulo sets of Lebesgue measure zero. The method of obtaining the system (3) from (2) is obtained and discussed.

\[\text{Y. N. Dowker}\]