The quantum theory of collision systems leads to functions $R(z)$ with the properties (P): $R(z)$ is defined, real-valued and differentiable for all $z > 0$, except for isolated singular points $Z_\nu$. The matrix

$$
\kappa_{ii} = R'(z_i), \quad \kappa_{ik} = \frac{R(z_i) - R(z_k)}{z_i - z_k}, \quad (i, k = 1, \ldots, n)
$$

is positive semi-definite for every $n$ and non-singular points $z_i$. It is shown that $R(z)$ is either continuous at a singularity $Z_\nu$ or tends to infinity on approach from either side. In the first case $Z_\nu$ is called an apparent singularity (a.s.), in the second case a real singularity (r.s.). $R(z)$ can be defined at an a.s. to be continuously differentiable there and to still satisfy the postulate (P). $S(z) = -R(z)^{-1}$ satisfies also the postulate (P) and if $Z_\nu$ is a r.s. for $R(z)$, it is an a.s. for $S(z)$ such that $S(Z_\nu)$ and $S'(Z_\nu)$ are well defined.

$$
R_\nu(z) = R(z) - \left[ S'(z_\nu)(Z_\nu - z) \right]^{-1}
$$

satisfies again (P) and has $Z_\nu$ as an a.s. only. This fact allows a development of $R(z)$ into a continued fraction with rational approximants and, hence, its analytic continuation into the complex plane. It is shown that $R(z)$ is meromorphic in the entire $z$-plane, slit along the negative axis; that is, has only simple poles, situated on the positive axis and with negative residues.

Wigner has studied previously so-called $R$-functions [e.g., Amer. Math. Monthly 59, 669–683 (1952); MR0051309] which are meromorphic in the entire $z$-plane and have a positive (negative) part in the upper (lower) half-plane, respectively. It is now shown that these $R$-functions satisfy the postulate (P) and that, conversely, each function $R(z)$ is a limit of such $R$-functions.

The close relation between the above theory and Löwner’s theory of monotonic matrix functions [Math. Z. 38, 177–216 (1934)] is pointed out.