MR0043386 (13,254a) 46.3X
von Neumann, Johann [von Neumann, John]
Eine Spektraltheorie für allgemeine Operatoren eines unitären Raumes.
(German)

Let $A$ be a bounded linear operator in the unitary space $U$ and let $S$ be a set of complex numbers. If for every rational function $f$ with $|f(\lambda)| \leq 1$ for $\lambda \in S$ we have $f(A)$ existing as an everywhere defined and bounded operator with bound $|f(A)| \leq 1$, then $S$ is said to be a spectral set of $A$. It is shown that the closed unit circle $|\lambda| \leq 1$ is a spectral set of $A$ if and only if $|A| \leq 1$. The proof makes use of the theory of $E$-functions introduced by I. Schur. Of the many interesting consequences we mention the following. If $\alpha$ is complex and $\beta > 0$ the set of all $\lambda$ with $|\lambda - \alpha| \leq \beta$ ($|\lambda - \alpha| \geq \beta$) is a spectral set of $A$ if and only if $|(A - \alpha I)| \leq \beta$ ($|(A - \alpha I)^{-1}| \leq \beta^{-1}$). The intersection of all spectral sets of $A$ is the spectrum of $A$. Either one of the following statements is necessary and sufficient for $A$ to be unitary. The set of $\lambda$ with $|\lambda| \leq 1$ and the set of $\lambda$ with $|\lambda| \geq 1$ are spectral sets of $A$. The set of $\lambda$ with $|\lambda| = 1$ is a spectral set of $A$. A similar result holds for hermitian operators. The operator $A$ is normal if and only if it has an absolute minimal spectral set. Another condition equivalent to normality is that the intersection of two spectral sets is a spectral set.

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