Let $T$ be an ergodic automorphism of a Lebesgue space $(X, \mu)$, $G$ a topological abelian group. Two measurable functions $f, g: X \to G$ are called $(T, G)$-homologous if there is a measurable function $\phi: X \to G$ such that $f(x) - h(x) = \phi(Tx) - \phi(x)$, the group operations in $G$ being written additively. This paper contains a number of results about the homology classes of functions. These theorems, roughly speaking, are of the following form: given an $f$ satisfying certain weak conditions, there is an $h$ in the homology class of $f$ satisfying stronger conditions. Most of the results concern the cases $G = \mathbb{R}$ and $G = \mathbb{Z}$, but there are some extensions to more general groups. We quote an interesting sample result: If $f \in L^1$ and $\int f = K$, then there is a function $h$, $(T, \mathbb{Z})$-homologous to $f$ which takes the values $K - 1, K, K + 1$ if $K$ is an integer, or $[K], [K] + 1$ if $K$ is not an integer.

One interpretation of the results in this paper is that they give different special representations of a special flow, or automorphism, over a given $T$. In this interpretation there are some interesting consequences [see A. B. Katok, Izv. Akad. Nauk SSSR Ser. Mat. 41 (1977), no. 1, 104–157].

{English translation: Soviet Math. Dokl. 17 (1976), no. 6, 1637–1641.}  

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