A method for the numerical calculation of hydrodynamic shocks.


The main object of this paper is a practicable numerical method for computing hydrodynamic flow involving shocks. Determination of the position of these shock discontinuities is part of the initial-boundary value problem, and this fact is a formidable obstacle to actual solution of problems. As suggested before, one could overcome the difficulty by replacing the system $S$ of equations, which is of the first order, by another system $S'$, of second order, which takes into account the factor $\lambda$ of heat conduction and the factor $\mu$ of viscosity. For $\lambda, \mu \to 0$ the system $S'$ tends to $S$, and it is plausible that the solutions of $S'$, all of which are continuous, tend in the limit to solutions of $S$, exhibiting the shock discontinuities in question. This remark immediately suggests numerical procedures which, however, appear rather complicated. In the present paper the authors have modified the attempt in a remarkable way. Again the system $S$ is replaced by another system $S''$, of second order and having continuous solutions; but the authors observe that the system $S''$ can be chosen in many ways, depending on one parameter, so that in the limit as this parameter tends to zero the system $S$ results and that the solutions of $S''$ tend to the solutions of $S$, exhibiting the proper shocks. The specific proposal in the paper is to replace in $S$ the pressure $p$ by an expression $p + q$, where $q$, containing the first derivative of the specific volume, is properly chosen. This replacement, while not having direct physical significance, lends itself to a simpler theoretical and numerical treatment than that involving heat conduction and viscosity. The paper discusses a simple special case. Furthermore, general schemes for finite difference procedures are set up and the stability of the computational procedures concerning these finite difference schemes is ascertained.

\textit{R. Courant}

\copyright\ Copyright American Mathematical Society 2018