Summary: “We show that the problem of deciding positivity of Kronecker coefficients is NP-hard. Previously, this problem was conjectured to be in P, just as for the Littlewood-Richardson coefficients. Our result establishes in a formal way that Kronecker coefficients are more difficult than Littlewood-Richardson coefficients, unless P = NP.

“We also show that there exists a #P-formula for a particular subclass of Kronecker coefficients whose positivity is NP-hard to decide. This is an evidence that, despite the hardness of the positivity problem, there may well exist a positive combinatorial formula for the Kronecker coefficients. Finding such a formula is a major open problem in representation theory and algebraic combinatorics.

“Finally, we consider the existence of the partition triples \((\lambda, \mu, \pi)\) such that the Kronecker coefficient \(k^{\lambda \mu, \pi} = 0\) but the Kronecker coefficient \(k^{l\lambda \mu, l\pi} > 0\) for some integer \(l > 1\). Such ‘holes’ are of great interest as they witness the failure of the saturation property for the Kronecker coefficients, which is still poorly understood. Using insight from computational complexity theory, we turn our hardness proof into a positive result: We show that not only do there exist many such triples, but they can also be found efficiently. Specifically, we show that, for any \(0 < \epsilon \leq 1\), there exists \(0 < a < 1\) such that, for all \(m\), there exist \(\Omega(2^{m^a})\) partition triples \((\lambda, \mu, \mu)\) in the Kronecker cone such that: (a) the Kronecker coefficient \(k^{\lambda \mu, \mu}\) is zero, (b) the height of \(\mu\) is \(m\), (c) the height of \(\lambda\) is \(\leq m^\epsilon\), and (d) \(|\lambda| = |\mu| \leq m^3\). The proof of the last result illustrates the effectiveness of the explicit proof strategy of GCT.”

References

8. P. Brgrisser, M. Christandl & C. Ikenmeyer (2011a). Nonvanishing of Kro-
30. K. Mulmuley (2010b). Geometric complexity theory VI: the flip via positiv-

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.