This paper, according to the authors, is Part III of their series of three papers [Part I, Comm. Pure Appl. Math. 69 (2016), no. 5, 981–1014; MR3481286; Part II, Calc. Var. Partial Differential Equations 55 (2016), no. 2, Art. 25; MR3465441] concerning the Lotka-Volterra competition-diffusion model. The authors further illustrate the general results obtained in Part I, while in Part III, they focus on the case when the two competing species have identical competition abilities and the same amount of total resources. In contrast to Part II, their results here show that in case both species have spatially heterogeneous distributions of resources, the outcome of the competition is independent of initial values but depends solely on the dispersal rates, which in turn depend on the distribution profiles of the resources, thereby extending the celebrated phenomenon “slower diffuser always prevails!” Furthermore, the species with a “sharper” spatial concentration in its distribution of resources seems to have the competitive advantage. Limiting behaviors of the globally asymptotically stable steady states are also obtained under various circumstances in terms of dispersal rates.

However, following Y. Lou’s approach, the authors considered in Part I the following heterogeneous system of PDEs in $\Omega \times \mathbb{R}_+$:

$$
\begin{align*}
U_t &= d_1 \Delta U + U(m_1(x) - U - cV), \\
V_t &= d_2 \Delta V + V(m_2(x) - bU - V).
\end{align*}
$$

In Parts II and III, they continue their investigation on this interesting and important system of PDEs. Especially, in the paper under review, they wish to clarify explicitly local and thus global dynamics for all $(d_1, d_2)$ in the special case $b = c = 1$, while both species have spatially heterogeneous distributions of resources. The results in Part I apply to various cases for $b$ and $c$; in particular, they include the case $bc \leq 1$ and encompass both “the slower diffuser always prevails” and a complete resolution of Lou’s conjecture on weak competition [see J. D. Dockery et al., J. Math. Biol. 37 (1998), no. 1, 61–83; MR1636644; Y. Lou, J. Differential Equations 223 (2006), no. 2, 400–426; MR2214941]. In Part II, they clarified explicitly local and thus global dynamics for all $(d_1, d_2)$ in the special case $b = c = 1$, when the spatial distribution of resources for one of the competing species is heterogeneous, while that of the other is uniform. Their results there showed that the species with homogeneous distribution of resources never prevails. Compared to Part II, the paper under review focuses on the case when both species have spatially heterogeneous distributions of resources. To be more precise, the authors consider the following heterogeneous system of PDEs with initial and homogeneous Neumann boundary conditions:

$$
\begin{align*}
U_t &= d_1 \Delta U + U(m_1(x) - U - V), \\
V_t &= d_2 \Delta V + V(m_2(x) - U - V), \\
\partial_\nu U = \partial_\nu V &= 0, \\
U(x, 0) &= U_0(x), V(x, 0) &= V_0(x)
\end{align*}
$$

in $\Omega \times \mathbb{R}_+$, where $m_1(x)$ and $m_2(x)$ satisfy the following condition (M): $m_1(x)$ and $m_2(x)$ are
nonnegative, nonconstant functions in \( C^\gamma(\Omega) \) \((\gamma \in (0, 1))\), \( m_1 \neq m_2 \) but \( \overline{m}_1 \equiv \overline{m}_2 \), where 
\[
\overline{m}_i = \frac{1}{|\Omega|} \int_\Omega m_i(x) \, dx, \quad i = 1, 2.
\]

Here, as in Parts I and II, \( U = U(x, t) \geq 0, V = V(x, t) \geq 0 \) represent the population densities of two competing species at location \( x \in \Omega \) and at time \( t > 0 \). \( \Omega \), the common habitat, is a bounded smooth domain in \( \mathbb{R}^N \) and \( d_1, d_2 > 0 \) are the dispersal rates of \( U \) and \( V \), respectively. The functions \( m_1(x) \) and \( m_2(x) \) represent the carrying capacities or intrinsic growth rates, which reflect the environmental influence on the species \( U \) and \( V \), respectively. The symbol \( \Delta \) represents the usual Laplace operator. The above model and its variants, in the past two decades, have attracted the attention of many researchers; see Part I and references therein.

The authors begin their discussion on the global dynamics of the above system by first examining an interesting phenomenon when both species are assumed to have identical heterogeneous distribution of resources; namely, “the slower diffuser always prevails!” By connecting the dynamics of the two systems studied in Part II and Part III here, they hope to have a clear understanding of the effects of spatial concentration on the global dynamics of the system.

Finally, the goal of this paper is to understand the competition outcome under the hypothesis (M). The results here give a quantitative description of the conclusion that the competition outcome is somewhat “comparable” to that of “slower diffuser always prevails” except that the co-existence now becomes a much stronger possibility in terms of dispersal rates.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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