Consi\ntion a random potential of the form
\[ V^\omega(x,t) = \sum_{k \in \mathbb{Z}} \sum_{i=1}^m \xi_k^{(i)}(x) \delta(t-k), \]
where \( \delta \) denotes the delta distribution, \( V_i \) are real-valued smooth functions on the \( \nu \)-dimensional torus \( T^\nu \) and \( (\xi_k^{(i)}(\omega))_{i=1}^m \) are identically distributed in \( \mathbb{R}^m \) with an absolutely continuous distribution, the authors study the forced Hamilton-Jacobi equation
\[ \partial_t \phi + \frac{1}{2} (\nabla_x \phi)^2 + V^\omega = 0 \]
for \( t \in \mathbb{R} \) and \( x \in T^\nu \). If \( \phi \) solves this equation, then \( \psi = \nabla_x \phi \) is a solution of the periodic Burgers equation
\[ \partial_t \psi + (\psi \cdot \nabla_x) \psi + \nabla_x V^\omega = 0. \]

The solutions \( \phi \) of the Hamilton-Jacobi equation, in turn, are related to orbits that minimize action integrals of an appropriate Lagrangian. In previous studies [W. E et al., Ann. of Math. (2) 151 (2000), no. 3, 877–960; MR1779561; R. Iturriaga and K. M. Khanin, Comm. Math. Phys. 232 (2003), no. 3, 377–428; MR1952472], the existence of unique globally minimizing orbits \( \zeta: \mathbb{R} \to T^\nu \) and global solutions—unique up to translations—have been established.

As the main result of the paper under review, the authors demonstrate that the unique global minimizer is hyperbolic (a.e.) and that the viscosity solutions are (not merely Lipschitz, but) smooth in a neighborhood of this global minimizer (where viscosity solutions refer to the solutions that are identical to the solutions of the viscous Burgers equation \( \partial_t \psi + (\psi \cdot \nabla_x) \psi + \nabla_x V^\omega = \lambda \nabla_x \psi \) in the limit as \( \lambda \to 0 \)).

These results are proved by a quantitative analysis connecting the variational problem for \( \zeta \) with the properties of Green bundles and by showing the transversality of the Green bundles and thus the nonvanishing of the Lyapunov exponents. To obtain the \( C^2 \) smoothness of the viscosity solutions in a neighborhood of the global minimizer, methods from the non-uniform hyperbolicity (Pesin) theory are invoked.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.