Special flows over interval exchange transformations (IETs) under roof functions with singularities at a subset of discontinuities of IET model area-preserving locally Hamiltonian flows on closed surfaces. The interval exchange comes from the Poincaré return map to a one-dimensional transversal and the roof function is given by the first return time. If the flow has only non-degenerate fixed points then the roof function has so-called symmetric logarithmic singularities.

In previous studies of the special flow representation, typical locally Hamiltonian flows on surfaces of genus $\geq 2$ with non-degenerate fixed points were shown to be weakly mixing [C. Ulcigrai, J. Mod. Dyn. 3 (2009), no. 1, 35–49; MR2481331], but not mixing [C. Ulcigrai, Ann. of Math. (2) 173 (2011), no. 3, 1743–1778; MR2800723; see also D. Scheglov, J. Mod. Dyn. 3 (2009), no. 1, 13–34; MR2481330].

The paper under review is concerned with the mild mixing property of special flows over IETs. Mild mixing is an intermediate property between weak mixing and mixing. Mild mixing was introduced into ergodic theory by H. Furstenberg and B. Weiss [in The structure of attractors in dynamical systems (Proc. Conf., North Dakota State Univ., Fargo, N.D., 1977), 127–132, Lecture Notes in Math., 668, Springer, Berlin, 1978; MR0518553], who showed that it is equivalent to absence of rigid factors; that is, a flow $\varphi^t: (X, \mu) \rightarrow (X, \mu)$ is mildly mixing if one cannot find a measurable set $A$ with $\mu(A) \in (0, 1)$ and a sequence of times $t_k \nearrow \infty$ such that $\mu(\varphi^{t_k}(A) \cap A) \rightarrow \mu(A)$, as $k \rightarrow \infty$.

The following is the main result of the paper.

Theorem. Special flows with symmetric logarithmic singularities over IETs of bounded type are mildly mixing.

IETs of bounded type are generalizations (via the Marmi-Moussa-Yoccoz cocyle) of rotations of bounded type. The proof of the theorem relies on the Fraczek-Lemańczyk criterion, which says that a weakly mixing, not partially rigid flow which satisfies the finite Ratner property is mildly mixing. Weak mixing follows from work of C. Ulcigrai. The authors show that being not partially rigid is implied by the bounded type condition on IETs. The heart of the proof is verification of the switchable Ratner property. This property is a weakening of the finite Ratner property, but is still sufficient to yield mild mixing. Thus this paper belongs to the stream of results which use various versions of Ratner’s property to enhance mixing of the flow.

As a corollary to their result the authors conclude that there exist locally Hamiltonian flows on surfaces of genus $\geq 2$ which are mildly mixing, but not mixing.

The paper is quite technical, but well written.

Andrey Gogolev

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.