A Besicovitch cylindrical transformation with a Hölder function.

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The author considers a cylindrical cascade $T_{\varrho,f}: \mathbb{T} \times \mathbb{R} \to \mathbb{T} \times \mathbb{R},$

$$T_{\varrho,f}(x,y) = (T_{\varrho}(x), y + f(x)), $$

where $T_{\varrho}: \mathbb{T} \to \mathbb{T}$ is a rotation of a circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, $T_{\varrho}(x) = x + \varrho$ (mod 1), and $f: \mathbb{T} \to \mathbb{R}$ is a continuous function. The so-called Besicovitch property [A. S. Besicovitch, Proc. Cambridge Philos. Soc. 47 (1951), 38–45; MR0039247] asserts: For any irrational circle rotation $T_{\varrho},$ there exists a continuous $f$ such that $T_{\varrho,f}$ is topologically transitive and has orbits that are closed sets (discrete orbits).

K. M. Frączek and M. Lemańczyk [Nonlinearity 23 (2010), no. 10, 2393–2422; MR2672680] constructed for almost every $\varrho$ a cylindrical transformation $T_{\varrho,f}$ satisfying the Besicovitch property with a $\gamma$-Hölder $f$ for $\gamma < \frac{1}{2}$ and, additionally, such that the Hausdorff dimension of the set of points in the circle having discrete orbits (the Besicovitch set) has the lower bound $1 - 2\gamma$. In this paper, the author constructs a cylindrical cascade with a $\gamma$-Hölder $f$ for any $\gamma \in (0,1)$.

The main result of the paper is as follows: For any $\gamma \in (0,1)$ and any $\varepsilon > 0,$ there exist a $\gamma$-Hölder $f$ and a circle rotation $T_{\varrho}$ such that the cylindrical transformation $T_{\varrho,f}$ satisfies the Besicovitch property and the Hausdorff dimension of the Besicovitch set in the circle is bigger than $1 - \gamma - \varepsilon.$ Several problems are formulated.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.