According to the authors, the present work is the first in a series of three papers concerning the Lotka-Volterra competition-diffusion model [Part II, X. He and W.-M. Ni, Calc. Var. Partial Differential Equations 55 (2016), no. 2, Art. 25; MR3465441]. More precisely, in this paper the authors investigate the combined effects of diffusion, spatial variation, and competition ability on the global dynamics of a classical Lotka-Volterra competition-diffusion system. They mainly study the following system of PDEs with initial and homogeneous Neumann boundary conditions:

\begin{align*}
U_t &= d_1 \Delta U + U(m_1(x) - U - cV) \quad \text{in } \Omega \times \mathbb{R}_+,
V_t &= d_2 \Delta V + V(m_2(x) - bU - V) \quad \text{in } \Omega \times \mathbb{R}_+,
\partial_\nu U &= \partial_\nu V = 0 \quad \text{on } \partial \Omega \times \mathbb{R}_+,
U(x,0) &= U_0(x), \quad V(x,0) = V_0(x) \quad \text{in } \Omega.
\end{align*}

Here, \( U = U(x,t) \geq 0 \) and \( V = V(x,t) \geq 0 \) represent the population densities of two competing species at location \( x \in \Omega \) and at time \( t > 0, \Omega \), the habitat, is a smooth bounded domain in \( \mathbb{R}^N \), and \( d_1, d_2 > 0 \) are the dispersal rates of \( U \) and \( V \), respectively; the species \( U \) and \( V \) have different distributions of resources or intrinsic growth rates. The functions \( m_1(x) \) and \( m_2(x) \) are \( C^\alpha(\overline{\Omega}), \alpha \in (0,1), \) with nonnegative average, and represent the carrying capacities or intrinsic growth rates which reflect the environmental influence on the species \( U \) and \( V \), respectively. The above model and its variants have attracted the attention of many researchers in the past two decades.

The authors establish the main results that determine the global asymptotic stability of semi-trivial as well as coexistence steady states. Hence, a complete understanding of the change in dynamics is obtained immediately. Their results indicate/confirm that, when spatial heterogeneity is included in the model, “diffusion-driven exclusion” could take place when the diffusion rates and competition coefficients of both species are chosen appropriately.

For the general \( 2 \times 2 \) Lotka-Volterra competition-diffusion system of the above form, under a very mild condition (see condition (M)), they completely classify, for some value of \( b, c, (b, c \in \Xi, \text{ see set } \Xi) \), its global dynamics in terms of the diffusion rates \( d_1, d_2 \).

As a special case, when the two species are competing for exactly the same resources with exactly the same growth rates, they establish Y. Lou’s conjecture [J. Differential Equations 223 (2006), no. 2, 400–426; MR2214941], which asserts that competition-exclusion becomes possible, even for weak competition, for appropriate diffusion rates if and only if at least one of the species’ competition ability exceeds a threshold value. Furthermore, their result in this special case connects Lou’s conjecture with the well-known phenomenon “the slower diffuser always prevails” and hence puts them in perspective. Their main results for general resources/growth rates seem to indicate that, as far as the distribution of resources is concerned, the more “heterogeneous” (measured by the quantity \( E(m), \text{ see relation (1.5)} \)), the better!

In conclusion, their method applies to an even more general competition system in
which the diffusion rates \(d_1, d_2\) are allowed to be spatially dependent, \(d_1 = d_1(x), d_2 = d_2(x) \in C^{1,\alpha}(\Omega)\), and both greater than some \(d_0 > 0\).

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References

15. He, X.; Ni, W.-M. Global dynamics of the Lotka-Volterra competition-diffusion system with equal amount of total resources, III. Preprint. MR3465441

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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