Averaging and trajectories of a Hamiltonian system, originating in graphene, placed in a strong magnetic field and a periodic electric field.

In this paper the authors study the motions in a Hamiltonian system with two degrees of freedom and Hamiltonian of the form

$$H = \varepsilon U(x) \pm (G(p, x, \varepsilon))^{1/2}, \quad G = (p_1 + x_2)^2 + p_2^2 + (\varepsilon m(x))^2,$$

where \( p = (p_1, p_2) \in \mathbb{R}^2 \) and \( x = (x_1, x_2) \in \mathbb{R}^2 \) are canonically conjugate variables, the functions \( U \) and \( m \) are analytic and periodic with respect to the lattice \( \Gamma \) generated by vectors \((2\pi, 0)\) and \((a, b)\) with \( b \neq 0 \), and \( \varepsilon > 0 \) is a small parameter. Such systems describe the classical motion of electrons and holes in graphene placed in a strong homogeneous magnetic field and an electric field with a periodic potential (the quantity \( 1/\varepsilon \) characterizes the strength of the magnetic field).

For an arbitrary \( E \in \mathbb{R} \), the authors introduce the new Hamiltonian

$$f_E = G/2 + \varepsilon U - (\varepsilon U)^2/2,$$

which is analytic for any \( \varepsilon \), including \( \varepsilon = 0 \). The key observation is that the energy level hypersurface \( \Sigma = \{ H = E \} \) coincides with the energy level hypersurface \( \{ f_E = E^2/2 \} \). According to the Maupertuis-Jacobi principle, the phase trajectories of \( H \) and \( f_E \) on \( \Sigma \) also coincide (but have different time parameterizations). Then the authors apply some celebrated results from Hamiltonian perturbation theory to explore the dynamics displayed by the Hamiltonian \( f_E \) on the energy level hypersurface \( \Sigma \).

Averaging theory provides new \( E \)-dependent symplectic coordinates \((J, y_1)\) and \((\phi, y_2)\) such that

$$f_E = \tilde{f}_E(J, y_1, y_2) + O(e^{-C/\varepsilon}),$$

where \( C > 0 \) is a certain constant, \( J \in \mathbb{R}, \phi \in \mathbb{R}/2\pi\mathbb{Z}, \) and \((y_1, y_2) \in \mathbb{R}^2/\Gamma \). The first step of the averaging procedure is described in detail. The Hamiltonian system with the truncated Hamiltonian \( \tilde{f}_E \) is completely integrable and determines a Hamiltonian system with one degree of freedom on \( \mathbb{T}^2 = \mathbb{R}^2/\Gamma \) for each value of \( J \). The trajectories on \( \mathbb{T}^2 \) are classified by means of Reeb graphs. As a particular example, the authors examine the case of \( U(x) = \cos x_1 + \cos \beta x_2 \) with \( \beta > 0 \).

Then KAM theory enables one to obtain many Lagrangian invariant 2-tori (carrying quasi-periodic motions) of the Hamiltonian system with the Hamiltonian \( f_E \) on \( \Sigma \). Finally, the authors show that the dynamics on these tori induced by the Hamiltonian system with the initial Hamilton function \( H \) is also quasi-periodic, and the corresponding frequencies are calculated.

Mikhail B. Sevryuk

References

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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