The paper under review is the second part of a series of three papers by the authors [Part I, Comm. Pure Appl. Math. 69 (2016), no. 5, 981–1014; MR3481286; Part III, “Global dynamics of the Lotka-Volterra competition-diffusion system with equal amount of total resources. III”, preprint, per bibl.] concerning the Lotka-Volterra competition-diffusion model. The authors focus on the joint effects of diffusion and spatial concentration on the global dynamics of a classical Lotka-Volterra competition-diffusion system. For comparison purposes, they assume that the two species have identical competition abilities and the same amount of total resources. In Part II, two kinds of resources are assumed: the case when the spatial distribution of resources for one species is heterogeneous while that of the other is homogeneous. In Part I, following Lou’s approach in [Y. Lou, J. Differential Equations 223 (2006), no. 2, 400–426; MR2214941], the authors considered the following system of PDEs in $\Omega \times \mathbb{R}_+$:

$$
\begin{align*}
U_t &= d_1 \Delta U + U(m_1(x) - U - cV), \\
V_t &= d_2 \Delta V + V(m_2(x) - bU - V).
\end{align*}
$$

In Parts II and III, the authors continue their investigation of this interesting and important system of PDEs. The results in Part I apply to various cases for $b$ and $c$; in particular, they include the case $bc \leq 1$ and encompass both “the slower diffuser always prevails” [J. D. Dockery et al., J. Math. Biol. 37 (1998), no. 1, 61–83; MR1636644] and a complete resolution of Lou’s conjecture on weak competition.

In the present paper, the authors wish to clarify explicitly local and thus global dynamics for all $(d_1, d_2)$ in the special case $b = c = 1$. They first take up the case when the spatial distribution of resources for one of the competing species is heterogeneous, while that of the other is uniform. Thus, the following system with initial and homogeneous Neumann boundary conditions is considered:

$$
\begin{align*}
U_t &= d_1 \Delta U + U(m(x) - U - V), & \text{in } \Omega \times \mathbb{R}_+, \\
V_t &= d_2 \Delta V + V(\overline{m} - U - V), & \text{in } \Omega \times \mathbb{R}_+, \\
\partial_n U &= \partial_n V = 0, & \text{on } \partial \Omega \times \mathbb{R}_+, \\
U(x, 0) &= U_0(x), & V(x, 0) = V_0(x) & \text{in } \Omega,
\end{align*}
$$

with $m_1(x) = m(x)$ and $m_2(x) = \overline{m} = \int_\Omega m(x)dx$, where $m(x) \in C^\alpha(\overline{\Omega})$ $(\alpha \in (0, 1))$, $m \neq \text{const}$ and $m \geq 0$ on $\overline{\Omega}$. Here, as in Part I, $U = U(x, t) \geq 0$ and $V = V(x, t) \geq 0$ represent the population densities of two competing species at location $x \in \Omega$ and at time $t > 0$, $\Omega$, the common habitat, is a bounded smooth domain in $\mathbb{R}^N$, and $d_1, d_2 > 0$ are dispersal rates of $U$ and $V$, respectively; the species $U$ and $V$ have different distributions of resources or intrinsic growth rates. The functions $\overline{m}$ and $m(x)$ represent the carrying capacities or intrinsic growth rates, which reflect the environmental influence on the species $U$ and $V$, respectively. The above model and its variants, in the past two decades, have attracted the attention of many researchers (see Part I and references therein).

Note that all intra- and inter-specific competition coefficients are normalized to 1, which means that the two species have identical competition abilities. Results on local
dynamics near semi-trivial steady states of the system were obtained in [X. He and W.-M. Ni, J. Differential Equations 254 (2013), no. 2, 528–546; MR2990042].

The authors’ goals include, first, completely clarifying the global dynamics of the above system in terms of \((d_1, d_2)\), and second, determining the asymptotic behaviors of coexistence steady states as well as the two semi-trivial steady states, when \(d_1\) and/or \(d_2\) tend to 0 or \(\infty\). Thus, in the present work, the results imply that in this case the former not only is guaranteed to survive, but will often wipe out the latter, regardless of initial values. Asymptotic behaviors of the stable steady states are also obtained for various limiting cases of the diffusion rates.

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References

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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