On dynamics of Lagrangian trajectories for Hamilton-Jacobi equations.
(English summary)

Characteristic curves of a Hamilton-Jacobi equation can be seen as action minimizing trajectories of fluid particles. The method of characteristics allows us to construct smooth solutions of first-order equations, and in general can be applied only locally. For non-smooth “viscosity” solutions, which give rise to discontinuous velocity fields, this picture holds only up to the moment when trajectories hit a shock and cease to minimize the Lagrangian action.

In this paper, the authors introduce the notions of admissible velocity and admissible momentum at a shock, and develop a local theory for Lagrangian particles in a gradient flow defined by a viscosity solution. Then, two physically meaningful regularization procedures are discussed brilliantly, one corresponding to vanishing viscosity and another to weak noise limit. The results show that, for any convex Hamiltonian, a viscous regularization allows us to construct a non-smooth flow that extends particle trajectories and determines dynamics inside the shock manifolds. This flow consists of integral curves of a particular “effective” velocity field, which is uniquely defined everywhere in the flow domain and is discontinuous on shock manifolds. The effective velocity field arising in the weak noise limit is generally non-unique and different from the viscous one, but in both cases there is a fundamental self-consistency condition constraining the dynamics.

References

9. CLARKE, F.H.: Optimization and Nonsmooth Analysis, Classics in Applied Math-


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