Stepin, A. M. [Stepin, Anatoli˘ı Mikha˘ılovich]
The connection between the approximative and spectral properties of metric automorphisms. (Russian)

Let \( T \) be an automorphism of the Lebesgue space \((X, \Sigma, \mu)\), \( U_T \) the induced unitary operator in \( L^2(\mu) \) and \( f: \mathbb{N} \to (0, +\infty) \). We say that \( T \) permits approximation by cyclic automorphisms (a.c.a.) of growth \( f(m) \) if there exist a sequence \( A_n = \{A_{n1}, \ldots, A_{np_n}\} \subset \Sigma \) of partitions of \( X \) and a sequence \( T_n \) of automorphisms of \((X, \Sigma, \mu)\) such that

\[
(1) \quad \lim_{n \to \infty} \inf_{1 \leq j \leq p_n} \mu(A_{nj} \Delta A) = 0
\]

for any \( A \in \Sigma \), (2) \( T_n \) permutes cyclicly the elements of \( A_n \), and (3) \( \sum_{j=1}^{p_n} \mu(T_n A_{nj} \Delta T A_{nj}) < f(n) \). The author’s main results are as follows. Theorem 1: If \( T \) permits a.c.a. of growth \( \theta/n \), \( \theta < 1 \), then the spectral type of \( U_T \) is singular. Theorem 2: If \( T \) permits a.c.a. of growth \( \theta/n \), \( \theta < 2 - 2/m \), then the spectral multiplicity of \( U_T \) is \( \leq m - 1 \). These theorems complete and improve results from §3 of the paper by A. B. Katok and the author [Uspehi Mat. Nauk 22 (1967), no. 5 (137), 81–106; MR0219697].

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