In this paper the authors deal with abstract first- and second-order identification problems. First, the Cauchy problem

\begin{align}
\begin{cases}
    u'(t) - Au(t) = f(t)z, & t \in [0, \tau], \\
    u(0) = u_0
\end{cases}
\end{align}

is considered in a Banach space \( X \), subject to the additional information \( \Phi(u(t)) = g(t) \) for any \( t \in [0, \tau] \) and some \( \Phi \in X^* \). The unknowns are \( u \) and the continuous function \( f \). As typical when dealing with identification problems, assuming that \( \Phi(z) \neq 0 \) and applying \( \Phi \) to both sides of the differential equation in (1) allows one to make \( f \) explicit in terms of \( u \). More precisely, it turns out that \( f = Bu = \phi(u)z \) for some \( \phi \in X^* \). Replacing the expression of \( u \) in the differential equation leads to a Cauchy problem associated with the operator \( A + B \). Here, the choice of \( z \) in \( F_1 \) is crucial. Indeed, it implies that \( A + B \) is the generator of a strongly continuous semigroup and this allows the authors to prove that, for any \( u_0 \in D(A) \), \( g \in W^{2,1}((0, \tau); C) \), satisfying the compatibility condition \( \Phi(u_0) = g(0) \), the identification problem admits a unique solution \( (u, f) \) with \( u \in C([0, \tau]; D(A)) \cap C^1([0, \tau]; X) \) and \( f \in C([0, \tau]; C) \).

The identification problem is also considered in the case when \( A \) is a sectorial operator, \( z \) belongs to the interpolation space \( D_A(\theta, \infty) \) and \( u_0 \in D_A(1 + \theta, \infty) \) for some \( \theta \in (0, 1) \). Depending on the smoothness of \( g \) two different existence-uniqueness results are established.

Next, the authors consider the complete second-order problem

\begin{align}
\begin{cases}
    u''(t) - Bu'(t) - Au(t) = f(t)z, & t \in [0, \tau], \\
    u(0) = u_0, & u'(0) = u_1,
\end{cases}
\end{align}

in the unknowns \( (u, f) \) still subject to the condition \( \Phi(u) = g \). Here, \( X \) is a Hilbert space, \( A = -C^*C \) for some densely defined invertible operator \( C \) and \( B \) is a dissipative operator whose domain contains \( D(C) \). Assuming that \( z \in D(C) \) satisfies \( \Phi(z) \neq 0 \), \( u_0 \in D(A) \), \( u_1 \in D(C) \), \( g \) is twice continuously differentiable and the compatibility conditions are satisfied, the authors prove the existence and uniqueness of a solution \( (u, f) \) to problem (2), such that \( \Phi(u) = g \), with \( u \in C^2([0, \tau]; X) \cap C([0, \tau]; D(A)) \), \( u' \in C([0, \tau]; D(B)) \), and \( f \in C([0, \tau]; C) \). Also, the case when the pair \((B, A)\) in (2) is replaced by \((2\gamma A, C - A^2)\), for some \( \gamma \in (0, 1) \), is considered when \( A \) is a sectorial operator with spectrum contained in \((-\infty, \omega)\) for some \( \omega < 0 \) and \( C \) is a suitable closed operator, whose domain contains the domain of \( A^2 \). Assuming that \( (C - A)y_0 \in D_A(\theta, \infty) \) for some \( \theta \in (0, 1) \), \( y_1 \in D_A(1 + \theta, \infty) \), \( z \in D_A(\theta, \infty) \) and \( g \in C^2([0, \tau]; X) \) satisfy due compatibility conditions, the analyticity of \( A \) allows the authors to show that \( u \) is smoother than in the general case.
Several applications of the abstract results are provided to population equations, to degenerate parabolic equations, to equations arising in the theory of linear visco-elastic materials and to damped wave and plate equations.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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