Ratner’s property for special flows over irrational rotations under functions of bounded variation. (English summary)


Special flows over the rotation by an irrational \( \alpha \) under roof functions \( f \) are considered. In [Ann. of Math. (2) 33 (1932), no. 3, 587–642; MR1503078] J. von Neumann proved that, for an arbitrary irrational \( \alpha \) and a piecewise absolutely continuous \( f \), von Neumann’s condition, \( \int f' \, d\lambda \neq 0 \), implies weak mixing of the corresponding special flow.

In the paper under review roof functions from the set \( \mathcal{U} \) are considered. These are functions of bounded variation without a continuous, singular part in the Lebesgue decomposition and whose sum of jumps are not equal to zero.

von Neumann’s result is generalized and it is shown that the special flow over the rotation by an arbitrary irrational \( \alpha \) with roof function in \( \mathcal{U} \) is weakly mixing.

With the additional assumptions that \( \alpha \) has bounded partial quotients and the roof function satisfies a stability condition under sufficiently small perturbations, the weak Ratner property of the special flow is also verified.

Under a suitable assumption about the jumps of the roof function with constants depending on the Diophantine approximation properties of \( \alpha \) it is also shown that the special flow is not partially rigid, which implies mild mixing of such flows.

The paper is a continuation of the results of [Ergodic Theory Dynam. Systems 26 (2006), no. 3, 719–738; MR2237466; J. Mod. Dyn. 4 (2010), no. 4, 609–635; MR2753947] in which K. M. Frączek and M. Lemańczyk proved that (the finite) Ratner property holds for each special flow given by an irrational rotation by \( \alpha \), with \( \alpha \) having bounded partial quotients, and the roof function piecewise absolutely continuous, satisfying von Neumann’s condition.

The weak Ratner property originates from the paper [K. M. Frączek and M. Lemańczyk, op. cit.; MR2753947].

Results and ideas from the above two papers are used in the proofs of the main theorems.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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