Stepin, A. M. [Stepin, Anatoli˘ı Mikha˘ılovich]

Dynamical systems on homogeneous spaces of semisimple Lie groups. (Russian)

Let $G$ be a Lie group, $\Gamma$ a closed subgroup of $G$ and $\mu$ a $G$-invariant measure on $G/\Gamma$ with $\mu(G/\Gamma) = 1$. An element $g \in G$ induces a transformation $T(g)$ of $G/\Gamma$ by $T(g)(g_0\Gamma) = gg_0\Gamma$ for $g_0 \in G$. Let $U$ be the unitary representation of $G$ on the representation space $L^2(G/\Gamma, \mu)$ defined by $(U(g)f)(x) = f(g^{-1}x)$ for $f \in L^2(G/\Gamma, \mu)$, $g \in G$ and $x \in G/\Gamma$.

Let $g$ be the Lie algebra of $G$ and $\exp : g \rightarrow G$ the exponential map. An element $x \in g$ is said to be ergodic if the flow $\{T(\exp tx)\}$ on $G/\Gamma$ is ergodic with respect to the measure $\mu$. Most of this paper is devoted to a proof of the following theorem: Let $G$ be a connected semisimple Lie group and $\Gamma$ a closed subgroup of $G$ with finite volume. If $x \in g$ is ergodic, then the flow $\{T(\exp tx)\}$ has countably multiple Lebesgue spectrum. In the proof of the above theorem, the author defines the notion of expanding [contracting] horospheric subgroup $H^+ [H^-]$ of $G$ with respect to $x \in g$, and gives a property of $H^\pm$.

Finally, the author defines a $K$-element $x \in g$ by requiring that for every $\Gamma$ for which $G/\Gamma$ has finite volume, $\{T(\exp tx)\}$ is a $K$-flow on $G/\Gamma$, and proves the following result: Let $G$ be semi-simple and let $g = \sum g_i$ be the direct sum decomposition into simple ideals $g_i$ of $g$; then $x = \sum x_i$ with $x_i \in g_i$ is a $K$-element if and only if the eigenvalues of $\text{ad} x_i$ have nonzero real parts.

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