This book-size paper is devoted to proving Kontsevich’s Homological Mirror Symmetry Conjecture for smooth Calabi-Yau hypersurfaces $M \subset \mathbb{CP}^{n-1}$ with symplectic form in the cohomology class $[\omega] = n^2 c_1(O(1))$ for $n \geq 5$ ($n = 3$ is the elliptic curve case due to Polishchuk and Zaslow, and $n = 4$ is the quartic surface case due to Seidel). The proof is rigorous modulo an unpublished result of Abouzaid et al. and a technical assumption about embedding of Fukaya categories (see below).

To give a precise statement denote by $\Lambda$ the universal Novikov ring, and by $\mathbb{C}[\![r]\!] \subset \Lambda$ the ring of formal power series in $r$. Let $\mathcal{F}(M)$ be the Fukaya category of $M$ ($\mathbb{Z}$-graded $\Lambda$-linear $A_\infty$ category), and let $D^\pi \mathcal{F}(M)$ be the split-closed derived Fukaya category ($\Lambda$-linear triangulated $A_\infty$ category). This is the $A$-model structure. The definition of $\mathcal{F}(M)$ and a criterion of split-generation for it rely on the work of Abouzaid et al.

On the $B$-side let $N$ be the mirror of $M$, and

$$\tilde{N}_{\text{nov}} := \{ u \in \mathbb{P}^{n-1}_\Lambda \mid u_1 \cdots u_n + r \sum_{j=1}^n u_j^2 = 0 \}.$$  

The group $(\mathbb{Z}_n)^n/(1, \ldots, 1)$ acts on $\mathbb{P}^{n-1}_\Lambda$ by multiplying coordinates by $n$-th roots of unity, and the author denotes by $\Gamma_n^* \subset \tilde{N}_{\text{nov}}$ the kernel of its homomorphism to $\mathbb{Z}_n$ given by summing the entries. Then the $B$-model structure is a DG enhancement of the bounded derived category of coherent sheaves on $N_{\text{nov}} := \tilde{N}_{\text{nov}}/\Gamma_n^*$, namely $D^b(\text{Coh}(N_{\text{nov}})) \cong D^b(\text{Coh}^{\Gamma_n^*}(\tilde{N}_{\text{nov}}))$. The main result then is the following.

Theorem: For $n \geq 5$ there exists a power series $\psi \in \mathbb{C}[\![r^n]\!]$ with $\psi(0) = \pm 1$ such that there is a quasi-equivalence of $\Lambda$-linear triangulated categories $D^\pi \mathcal{F}(M) \cong \hat{\psi}^* \cdot D^b(\text{Coh}(N_{\text{nov}}))$, where $\hat{\psi}^*$ is a lift to $\Lambda$ of the automorphism $\psi^*$ of $\mathbb{C}[\![r]\!]$ induced by $\psi$.

On the $A$-side the key technical construction introduced by the author is the relative Fukaya category $\mathcal{F}(M, D)$, where $D \subset M$ is a smooth normal crossings divisor. Its objects are compact, exact, embedded, anchored Lagrangian branes in $M \setminus D$, and $A_\infty$ structure maps count rigid boundary punctured holomorphic disks in $M$. One can think of it as a graded deformation of $\mathcal{F}(M \setminus D)$, but with a non-traditional grading structure. The aforementioned technical assumption is that for a certain coefficient ring $R$ the category $\mathcal{F}(M, D) \otimes_R \Lambda$ fully faithfully embeds into $\mathcal{F}(M)$. To study the behavior of $\mathcal{F}(M, D)$ for branch covers the author also introduces the relative smooth orbifold Fukaya category, which tracks degrees of ramification of the cover over the branches. Explicit computations employ a Morse-Bott model of $\mathcal{F}(M, D)$ based on the ‘cluster homology’ of Cornea and Lalonde. On the $B$-side the main computational tool is the category of graded matrix factorizations, which serves as the mirror to the relative Fukaya category.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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