In this article, the authors consider horocycle flows on the unit tangent bundle of a surface of constant negative curvature and prove a number of limit theorems for the ergodic average of functions under the horocycle flow.

The results are similar in spirit to the one obtained on translation flows on flat surfaces in [A. I. Bufetov, Ann. of Math. (2) 179 (2014), no. 2, 431–499; MR3152940], as well as those on area-preserving flows in [G. Forni, Ann. of Math. (2) 155 (2002), no. 1, 1–103; MR1888794; Ann. of Math. (2) 146 (1997), no. 2, 295–344; MR1477760].

The methods, however, are different. The authors introduce some finitely additive Hölder measures on rectifiable arcs on the surface, invariant under one of the horocycle flows, and show that they are in one-to-one correspondence with a certain subspace of invariant distributions for the other horocycle flow. They then rely on the classification of distributions invariant under a horocycle flow obtained in [L. Flaminio and G. Forni, Duke Math. J. 119 (2003), no. 3, 465–526; MR2003124].

The main results are too involved to be precisely formulated here. To give a flavor, in the nicest case, when the smallest eigenvalue $\mu_0$ of the Laplace operator on the surface is strictly less than $1/4$, the variance of the ergodic average up to time $T$ of a smooth function grows at the rate of $T^{(1+\nu_0)/2}$, where $\nu_0 = \sqrt{1-4\mu_0}$. Once the ergodic average is normalized to have variance 1, it converges in distribution to a nondegenerate compactly supported measure on the real line.

References

10. S. G. Dani, Invariant measures and minimal sets of horospherical flows, Invent.


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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