The Fried average entropy and slow entropy for actions of higher rank abelian groups. (English summary)


The authors consider two entropy-type invariants for smooth measure-preserving actions of $\mathbb{Z}^k$ (although a general discussion is made for actions of $\mathbb{R}^k$ to which, via suspension, actions of $\mathbb{Z}^k \times \mathbb{R}^n$ can be reduced):


The authors consider actions of higher abelian groups, that is, of $\mathbb{Z}^k$, $k \geq 2$, on $(k+1)$-dimensional manifolds. For $\mathbb{Z}^k$ actions and $1 \leq l \leq k$ it is possible to define the $l$-entropy based on Fried’s approach. The authors mainly pay attention to Cartan actions, i.e., ergodic automorphisms of the torus, since they essentially provide the universal model for positive Fried average entropy maximal rank actions of $\mathbb{Z}^k$, $k \geq 2$ (see the measure rigidity theorem in [A. Katok and F. Rodriguez Hertz, “Arithmeticity of maximal rank smooth abelian actions”, preprint; per bibl.]).

The authors show that the Fried average entropy is bounded from below by a positive constant, they obtain a good lower estimate for that constant, and they prove that the lower bound grows exponentially with the rank of the Cartan action. Moreover, lower bounds (polynomial and exponential) are found for many intermediate entropies. After the rigidity theorem, similar conclusions follow for weakly mixing actions of $\mathbb{Z}^k$ with positive Fried average entropy on a $(k+1)$-dimensional manifold for $k \geq 2$.

Then the authors prove that the slow entropy for a Cartan action on the torus $T^n$ depends only on the dimension and the value of the Fried average entropy, and that it is uniformly bounded away from 0. The lower bound grows linearly with the dimension. By the measure rigidity theorem, it follows that the same is true for any weakly mixing action of $\mathbb{Z}^k$ with positive slow entropy on a $(k+1)$-dimensional manifold for $k \geq 2$.

Thus, under the hypothesis of weak mixing, either both invariants vanish, or their values are bounded away from 0 by universal constants. Dropping the weak mixing assumption, it is possible to construct examples with arbitrarily small but positive Fried average entropy and slow entropy.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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