On asymptotic contractions. (English summary)

A constructive generalization of the Banach contraction mapping principle to the case of mappings with arbitrary rate of contraction is given.

In Section 2, the authors define the boundedness \((b_1)\) and an asymptotic contraction \((b_2)\) for selfmaps of a metric space.

**Theorem 1.** A continuous bounded asymptotic contraction \(F: U \to U\) on a complete metric space \(U\) has a unique fixed point \(z = F(z)\), and the iteration process \(u_n = F(u_{n-1})\) converges to \(z\) for an arbitrary initial approximation \(u_0 \in U\). Moreover, one has the estimate of the rate of attraction \(F^n(u_0)\) to \(z\).

In Theorem 2, a sufficient condition \((a_1) + (a_2)\) for continuous bounded asymptotic contractions is given. Theorem 3 states that \((a_1) + (a_2)\) is equivalent to \((b_1) + (b_2)\) for any continuous selfmapping of a complete metric space.

Section 3 contains examples and remarks, and the authors state that Theorems 1–3 specify a convergence criterion for the mappings satisfying conditions \((a_1)\) and \((a_2)\), which permits one to study the convergence for this class of problems in a unified way.

**References**


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*