Let $T$ be a circle homeomorphism and $\tau: \mathbb{R} \to \mathbb{R}$ a lift of $T$ (here the circle is $\mathbb{R}/\mathbb{Z}$). An orientation preserving circle homeomorphism $T$ is a circle map with a break when

1. there exists a point $y \in [0, 1)$ such that $\tau$ restricted to the interval $[y, y+1]$ is of class $C^r$, $r \in [1, \infty) \cup \{\infty, w\}$ (here $C^w$ corresponds to functions $\tau$ whose restriction to the interval $[y, y+1]$ has analytic extension on a complex disc containing $[y, y+1]$),
2. $\tau'(x) > c > 0$ for every $x \in [y, y+1]$, and
3. $\tau_+(y)$ and $\tau_-(y)$ denote the one-sided derivatives of $\tau$ at $y$ and are such that $\sqrt{\tau_-(y)/\tau_+(y)} = c$ with $c \in \mathbb{R}^+ \setminus \{1\}$.

The number $c$ is called the size of the break. The space of $(C^r)$ circle maps with break of size $c$ is denoted by $B^r_c$. In this paper the authors give examples of circle maps with break $T_\rho, T_\rho^w \in B^w_c$, with the same irrational rotation number $\rho$, and the same break size $c$ that cannot be conjugated by any Lipschitz continuous map. They also give examples of circle maps with break $T_\rho, T_\rho^w \in B^w_c$, for a class of irrational rotation numbers where the topological conjugacy is not $C^{1+\alpha}$, for any $\alpha > 0$. In this class of rotation numbers, K. M. Khanin and O. Yu. Teplinsky [Comm. Math. Phys. 320 (2013), no. 2, 347–377; MR3053764] proved that $C^1$-rigidity holds. It follows that these examples are of a different kind than those presented before.

References

10. A. Denjoy, Sur les courbes définies par les quations différentielles la surface du tore,

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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