In this paper, the authors present an almost complete renormalization theory for circle homeomorphisms which are $C^{2+\alpha}$-smooth ($\alpha > 0$) everywhere except for one break point. This renormalization is the main tool in establishing the rigidity result; this later means that under certain conditions, the smoothness of a conjugacy between circle homeomorphisms holds. The main step in proving rigidity is convergence of renormalizations. It was known [K. M. Khanin and E. B. Vul, in Dynamical systems and statistical mechanics (Moscow, 1991), 57–98, Adv. Soviet Math., 3, Amer. Math. Soc., Providence, RI, 1991; MR1118158; Y. Katznelson and D. S. Ornstein, Ergodic Theory Dynam. Systems 9 (1989), no. 4, 643–680; MR1036902] that renormalizations converge to a two-parameter family of linear-fractional maps. This essentially reduces the analysis to a study of renormalizations for this canonical family. The authors prove that, under certain conditions on the rotation numbers, the corresponding two-dimensional transformation has strong hyperbolic properties which allow them to construct the full renormalization horseshoe. The following $C^1$-rigidity result (which was first announced by the authors in [Invent. Math. 169 (2007), no. 1, 193–218; MR2308853]) is then derived from the convergence of renormalizations by means of the conditional theorem proved in [O. Yu. Teplinsky and K. M. Khanin, Uspekhi Mat. Nauk 59 (2004), no. 2(356), 137–160; MR2086641]:

The rigidity theorem. Two $C^{2+\alpha}$-circle ($\alpha > 0$) homeomorphisms smooth everywhere except for one break point with the same size of break and the same irrational rotation number of a half-bounded type (which include non-Diophantine numbers with arbitrarily high rate of growth) are $C^1$-smoothly conjugate.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.