In this work the authors study the interaction between dispersal and spatial variation in the classical Lotka-Volterra competition-diffusion system,

\[
\begin{align*}
U_t &= d_1 \Delta U + U(m_1(x) - U - cV), \quad x \in \Omega, \quad t > 0, \\
V_t &= d_2 \Delta V + V(m_2(x) - bU - V), \quad x \in \Omega, \quad t > 0, \\
\partial_n U &= \partial_n V = 0, \quad x \in \partial \Omega, \quad t > 0, \\
U(x,0) &= U_0(x), \quad V(x,0) = V_0(x), \quad x \in \Omega.
\end{align*}
\]

Here, \(U(x,t)\) and \(V(x,t)\) represent the population densities of two competing species in a bounded domain \(\Omega\), with migration rates \(d_1\) and \(d_2\) and carrying capacities \(m_1(x)\) and \(m_2(x)\), respectively. The constants \(b\) and \(c\) denote their inter-specific competition coefficients, while the intra-specific competition coefficients have been normalized to 1.

In this context, zero Neumann boundary condition means that no individual crosses the boundary of the habitat.

The authors analyze the dynamics of system (1). After recalling some previous results concerning specific values of such coefficients, they move to a more general framework and incorporate into the study different competition abilities and general carrying capacities.

In order to perform the study, a parameter \(\beta\) multiplying the function \(m_2(x)\) in the second equation is introduced. This parameter allows the authors to compare the intrinsic growth rates of the populations. They show that both extinction and coexistence of both species arise for different choices of the parameter and the coefficients, characterizing some of them where linear stability of one of the semi-trivial steady states holds and another where a co-existence steady state is expected.

As a consequence of the analysis, the authors also characterize the change of dynamics, as \(\beta\) increases from 0 to \(\infty\), near the two semi-trivial steady states. In particular, the results show how, for general \(m_1, m_2, b\) and \(c\), the three subsets of parameters \((d_1,d_2)\) involved in the stability analysis (parameters where each of the semi-trivial steady state solutions is linearly stable and where both semi-trivial solutions are linearly unstable) can evolve in a complicated way when \(\beta\) increases from 0 to \(\infty\).

{For Part I see [X. He and W.-M. Ni, J. Differential Equations 254 (2013), no. 2, 528–546; MR2990042].}
10. Y. Lou, private communication.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.