On the hyperbolicity of minimizers for 1D random Lagrangian systems. (English summary)

In this paper, the authors study minimizers of a random Lagrangian functional of the form
\[ A(\gamma) := \frac{1}{2} \int_s^t (\gamma'(\tau) - b)^2 d\tau + F(\gamma(\cdot), \gamma'(\cdot); s, t, \omega), \]
where \( \gamma: [s, t] \to S^1 \) is absolutely continuous, \( b \in \mathbb{R} \) is given, and \( F(\cdot, \cdot; \cdot, \cdot, \omega) \) is a random functional satisfying the so-called separation property. The authors define a separation property and then construct functionals satisfying this property. For a given \( x \in S^1 \), \( [s, t] \subset \mathbb{R} \), and \( \psi: S^1 \to \mathbb{R} \), \( \gamma^x_{s, t, \psi}: [s, t] \to S^1 \) denotes a minimizer of \( A(\gamma) + \psi(\gamma(s)) \) over all absolutely continuous curves \( \gamma: [s, t] \to S^1 \) such that \( \gamma(t) = x \). For \( -\infty < r < s \leq t < \infty \), let
\[ \Omega_{r, s, t, \psi} := \{ \gamma^x_{r, t, \psi}(s): x \in S^1 \}. \]

The sets \( \Omega_{r, s, t, \psi} \) are closed in \( S^1 \) and \( \Omega_{r, s, t, \psi} \subseteq \Omega_{r, s, t, \psi} \) when \( s \leq t_1 \leq t_2 \). For a given closed subset \( Z \subset S^1 \), let \( m(Z) \) denote the maximal length of a connected component of \( S^1 \setminus Z \), and let \( d(Z) := 1 - m(Z) \). The main result of the paper is:

Theorem. If the separation property for the functional \( F(\cdot; \cdot; \cdot, \cdot, \omega) \) holds, then there exist constants \( \lambda, B > 0 \) such that
\[ E(d(\Omega_{r, s, t, \psi})) \leq B \exp(-\lambda(t - s)) \]
for all \( -\infty < r < s \leq t < \infty \), where \( E(\cdot) \) denotes the expectation.  

References


Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

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