On the hyperbolicity of minimizers for 1D random Lagrangian systems. (English summary)


In this paper, the authors study minimizers of a random Lagrangian functional of the form
\[
A(\gamma) := \frac{1}{2} \int_s^t (\gamma'(\tau) - b)^2 d\tau + F(\gamma(s), \gamma'(s); s, t, \omega),
\]
where \(\gamma: [s, t] \to S^1\) is absolutely continuous, \(b \in \mathbb{R}\) is given, and \(F(\cdot, \cdot; \cdot, \cdot, \omega)\) is a random functional satisfying the so-called separation property. The authors define a separation property and then construct functionals satisfying this property. For a given \(x \in S^1\), \([s, t] \subset \mathbb{R}\), and \(\psi: S^1 \to \mathbb{R}\), \(\gamma^{x, s, t, \psi}: [s, t] \to S^1\) denotes a minimizer of \(A(\gamma) + \psi(\gamma(s))\) over all absolutely continuous curves \(\gamma: [s, t] \to S^1\) such that \(\gamma(t) = x\). For \(-\infty < r < s \leq t < \infty\), let
\[
\Omega_{r, s, t, \psi}^x := \{\gamma^{x, s, t, \psi}(s): x \in S^1\}.
\]
The sets \(\Omega_{r, s, t, \psi}^x\) are closed in \(S^1\) and \(\Omega_{r, s, t, \psi, t} \subseteq \Omega_{r, s, t, \psi, s}\) when \(s \leq t_1 \leq t_2\). For a given closed subset \(Z \subset S^1\), let \(m(Z)\) denote the maximal length of a connected component of \(S^1 \setminus Z\), and let \(d(Z) := 1 - m(Z)\). The main result of the paper is:

Theorem. If the separation property for the functional \(F(\cdot, \cdot; \cdot, \cdot, \omega)\) holds, then there exist constants \(\lambda, B > 0\) such that
\[
E(d(\Omega_{r, s, t, \psi}^{x})) \leq B \exp(-\lambda(t - s))
\]
for all \(-\infty < r < s \leq t < \infty\), where \(E(\cdot)\) denotes the expectation.

References


Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.