The classical horocycle flow on a compact surface of constant negative curvature is a fundamental example of parabolic flow. In contrast to the theory of hyperbolic flows, not much is known about the dynamical properties of general smooth parabolic flows, not even for smooth perturbations of classical horocycle flows. It would be very interesting to know to what extent the well-studied dynamical properties of the horocycle flow persist after a smooth time change. The paper under review investigates these problems.

The most important result available to date is due to B. H. Marcus and shows that all time changes satisfying a mild differentiable condition are mixing [Ann. of Math. (2) 105 (1977), no. 1, 81–105; MR0458496]. Marcus’ results generalized earlier work by A. G. Kushnirenko [Vestnik Moskov. Univ. Ser. I Mat. Meh. 29 (1974), no. 1, 101–108; MR0353369].

A. Katok and J.-P. Thouvenot, and also Kushnirenko in earlier work, conjectured that any flow that is a sufficiently smooth time change of a horocycle flow has countable Lebesgue spectrum. In reply, the authors prove in the paper under review sharp bounds on the rate of equidistribution and mixing of smooth time changes of the classical horocycle flow on the unit tangent bundle of a compact hyperbolic surface. Then they derive results on the spectrum of smooth time changes and prove that the spectrum is absolutely continuous with respect to the Lebesgue measure and that the maximal spectral type is indeed equivalent to Lebesgue.

Marcus’ argument is based on the equidistribution of arcs that are long in the horocycle direction and bounded in the complementary directions. Recent sharp results on the rate of equidistribution of such arcs were obtained in [A. Bufetov and G. Forni, “Limit theorems for horocycle flows”, preprint, arXiv:1104.4502]. Using in addition a bootstrap trick, the authors are able to prove that the decay of correlations of the horocycle flows is indeed stable under any smooth time change.

Spectral results are then derived from sharp bounds on the decay of correlations of smooth functions. The decay of correlations and the bootstrap trick are also crucial in the proof that the maximal spectral type is Lebesgue.

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