Let $T$ be an automorphism of a Lebesgue space $(X, \mu)$, $\Gamma$ a topological group, and $\Gamma(X)$ the set of measurable functions from $X$ to $\Gamma$. The transformation $T_f$ of $X \times \Gamma$ defined by $T_f(x, y) = (Tx, f(x)y)$ is said to be a $\Gamma$-extension of $T$. If $T$ is ergodic and $\Gamma = \mathbb{Z}_2$ it is shown that an ergodic $\Gamma$-extension $T_f$ always exists. This is equivalent to the equation $\phi(Tx) = f(x)\phi(x)$ having no solution $\phi$ in $\Gamma(X)$, and answers in the negative a question of P. R. Halmos [Lectures on ergodic theory, Math. Soc. Japan, 1956; MR0097489; reprint, Chelsea, New York, 1960; MR0111817]. The author next considers groups of automorphisms of Lebesgue spaces. If $P$ and $Q$ are countable groups of automorphisms of $X$ that are trajectory equivalent, then there is a one-to-one correspondence between the cohomology groups $H^1(P, \Gamma(X))$ and $H^1(Q, \Gamma(X))$, which is a group isomorphism if $\Gamma$ is commutative. As a corollary, the cohomology groups of all countable approximately finite [H. A. Dye, Amer. J. Math. 81 (1959), 119–159; MR0131516] ergodic groups of automorphisms are isomorphic.

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