The Beilinson-Drinfeld Grassmannian and symplectic knot homology. (English summary)

Kamnitzer, Joel (3-TRNT)


In [Duke Math. J. 134 (2006), no. 3, 453–514; MR2254624] P. Seidel and I. Smith used symplectic geometry to construct an invariant of links closely related to the SL₂ Khovanov homology of [M. G. Khovanov, Duke Math. J. 101 (2000), no. 3, 359–426; MR1740682]. They started with a space of matrices smoothly fibring over the configuration space of their distinct eigenvalues in C. The monodromy of this symplectic fibration gives a representation of the braid group in the symplectomorphism group of a fibre. Therefore the Floer cohomology between a certain canonical Lagrangian submanifold of the fibre and its image under elements of the braid group defines a braid invariant. Via a careful study of the singularities and vanishing cycles as two or three eigenvalues come together, the author proves that this braid invariant is also invariant under the Markov moves, and so is in fact an invariant of the link closure of the braid.


In the paper under review, the author constructs a symplectic fibration starting with any complex reductive group G. This unifies the previous ad hoc constructions: he shows that for G = SL₂ and SLₙ, it reproduces the spaces used by Seidel and Smith and Manolescu respectively. Those spaces can be described in terms of Higgs bundles on ℙ¹; Kamnitzer’s are closely related spaces of Hecke modifications of the trivial G-bundle on ℙ¹.

The author shows how the geometric Satake correspondence and the factorisation property of Beilinson-Drinfeld Grassmannians quickly imply the appropriate generalisations of the technical results of Seidel and Smith and Manolescu. Therefore he expects to be able to use Floer cohomology to construct link invariants from any complex reductive group G. This is left for future work, as is the mirror construction in algebraic geometry [cf. S. Cautis and J. Kamnitzer, Invent. Math. 174 (2008), no. 1, 165–232; MR2430980].

Richard P. Thomas

© Copyright American Mathematical Society 2018