Perfect retroreflectors and billiard dynamics. (English summary)

The exact velocity reversal (EVR) property of an open billiard domain means that the velocity of every incoming particle is reversed when the particle eventually leaves the domain. The authors construct a family of domains $D_{\epsilon,\sigma}$ for which EVR holds up to a set of initial conditions whose measure tends to zero if $\epsilon \to 0$. The domain $D_{\epsilon,\sigma}$ is an infinite tube $[0, \infty) \times [0, 1]$ with vertical barriers of height $\epsilon/2$ at the points $(\sigma n, 0)$ and $(\sigma n, 1)$ for every natural number $n$, where $\sigma > 0$ is the spacing of the barriers. The particle enters the tube at $x_0 = 0$ with initial velocity $v_0$ of absolute value 1. The phase space $\Omega = [0, 1] \times [-1/2, 1/2]$; the second coordinate stands for $\varphi/\pi$, where $\varphi$ is the angle of the velocity. The randomness comes from an arbitrary initial measure on $\Omega$, absolutely continuous with respect to the Lebesgue measure. The authors extend the semi-infinite tube to a bi-infinite one, and define the discrete time stochastic process $\xi^k_{\epsilon,\sigma}$ as the $x$ coordinate of the particle at the moment of the $k$-th reflection from the vertical wall. The paper states that $\xi^k_{\epsilon,\sigma}$ has a limit distribution, independent of the initial measure and of $\sigma$ when $\epsilon \to 0$. An analogous statement holds for the continuous time version of $\xi^k_{\epsilon,\sigma}$.

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*