Carleman estimates are derived for strongly parabolic, scalar operators in divergence form, with discontinuous coefficients. The coefficients are piecewise smooth, but may jump across a single interface. The interface is assumed smooth and not touching the boundary of the domain where the operator is defined. No assumptions are needed on the sign of the jump in the coefficients at the interface.

The first type of Carleman estimate is global in time and local in space in a neighborhood of the interface. The estimate is in terms of a non-smooth weight function that is assumed to satisfy a certain sub-ellipticity condition. A Weyl-Hörmander pseudodifferential calculus is employed to account for the different behavior of the operator with respect to time and space.

The proof is based on a certain decomposition in frequency into three distinct regions using microlocal cut-offs: a region of low frequencies in tangential directions, a region of high frequencies in tangential directions, and an intermediate region. Tangential operators are defined and associated calculi are developed to handle the low and high frequency regions. Both calculi are used in the intermediate region. In the low frequency region, the Carleman method can be used directly since trace terms obtained by integration by parts of the operator conjugated with the weight can be efficiently estimated on each side of the interface. In the high frequency region, a parametrix for the parabolic operator is exploited instead.

The local Carleman estimates are finally patched together to yield a global in space and time Carleman estimate again with non-smooth weight across the interface.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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