von Neumann, Ville, and the minimax theorem. (English summary)


Summary: “von Neumann proved the minimax theorem (existence of a saddle-point solution to 2 person, zero sum games) in 1928. While his second article on the minimax theorem, stating the proof, has long been translated from German, his first announcement of his result (communicated in French to the Academy of Sciences in Paris by Borel, who had posed the problem settled by von Neumann’s proof) is translated here for the first time. The proof presented by von Neumann and Morgenstern (1944) is not von Neumann’s rather involved proof of 1928, but is based on what they called ‘The Theorem of the Alternative for Matrices’, which is in essence a reformulation of an elegant and elementary result by Borel’s student Jean Ville in 1938. Ville’s argument was the first to bring to light the simplifying role of convexity and to highlight the connection between the existence of minimax and the solvability of systems of linear inequalities. It by-passes nontrivial topological fixed point arguments and allows the treatment of minimax by simpler geometric methods. This approach has inspired a number of seminal contributions in convex analysis including fixed point and coincidence theory for set-valued mappings. Ville’s contributions are discussed briefly and von Neumann’s original communication, Ville’s note, and Borel’s commentary on it are translated here for the first time.”

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