Let $G$ be a simply connected solvable Lie group with a discrete uniform subgroup $D$. Haar measure on $G$ induces a finite measure $\mu$ on $D\backslash G$ which is invariant under the action of the group $G$ defined by $S(g_0): Dg \to Dgg_0$. In this case the one-parameter subgroup $g_t$ of the group $G$ defines a flow in the space $(D\backslash G, \mu)$. Let $L_2(D\backslash G, \mu)$ be the Hilbert space of functions defined on $D\backslash G$ which are square-integrable with respect to the measure $\mu$. Let $U(g)$ be a unitary operator on $L_2(D\backslash G, \mu)$ associated with the transformation $S(g)$, $\mathcal{L}(G)$ the Lie algebra of $G$ and $\exp(\mathcal{L}(G))$ the exponential mapping of $\mathcal{L}(G)$ to $G$. Adapting these notations the author proves the following main theorem: If $X$ is a regular element of $\mathcal{L}(G)$, the flow $S(\exp tX)$ is ergodic and $\exp(\mathcal{L}(G))$ is a mapping onto the whole group, then $L_2$ has a decomposition into the direct sum of the subspaces $H_d$ and $H_\lambda$, which are invariant under the one-parameter group of the unitary operators $U(\exp tX)$ which have the property that the spectrum of the group $U(\exp tX)$ is discrete in the subspace $H_d$ but is a countable Lebesgue spectrum in the subspace $H_\lambda$. The case of the nilpotent group $G$ is also studied. For the notations and definitions, see L. Auslander, L. Green and F. Hahn [Flows on homogeneous spaces, Ann. of Math. Studies, No. 53, Princeton Univ. Press, Princeton, N.J., 1963; MR0167569; Russian translation, Izdat. “Mir”, Moscow, 1966; MR0213468].

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