Wehrheim, Katrin (1-MIT); Woodward, Chris T. [Woodward, Christopher T.] (1-RTG)

Functoriality for Lagrangian correspondences in Floer theory. (English summary)

A Lagrangian correspondence between symplectic manifolds $M_0$ and $M_1$ is a Lagrangian submanifold of the product $M_0^- \times M_1$, where $M_0^-$ denotes $M_0$ with the sign of its symplectic form reversed. The authors define a generalized Lagrangian submanifold of a symplectic manifold $M$ as a sequence of Lagrangian correspondences which starts with a one-point space and ends with $M$ and associates to $M$ (which is assumed compact and monotone or exact and geometrically bounded) a category $D^#_0(M)$ with objects consisting of generalized (compact, oriented, relatively Spin, monotone or exact) Lagrangian submanifolds and with morphisms quilted Floer homology classes (see [K. Wehrheim and C. T. Woodward, Geom. Topol. 14 (2010), no. 2, 833–902; MR2602853] for definitions and properties of quilted Floer homology). They show (under certain monotonicity conditions) that a Lagrangian correspondence $L_{01}$ between symplectic manifolds $M_0$ and $M_1$ induces a functor between $D^#_0(M_0)$ and $D^#_0(M_1)$ and furthermore that compositions of such functors agree with A. D. Weinstein’s geometric composition [in Differential geometric methods in mathematical physics (Clausthal, 1980), 45–51, Lecture Notes in Math., 905, Springer, Berlin, 1982; MR0657441] when the latter is defined.

The authors then present a somewhat stronger version of the above result, phrased in the language of 2-categories, which is useful for applications in low-dimensional topology. In particular, it leads to $SU(N)$ Floer theoretic invariants of 3-manifolds that can be thought of as the Lagrangian versions of gauge theoretic invariants of Donaldson and Floer, in the case without knots, and of P. B. Kronheimer and T. S. Mrowka [“Knot homology groups from instantons”, preprint, arxiv.org/abs/0806.1053] and O. Collin and B. Steer [J. Differential Geom. 51 (1999), no. 1, 149–202; MR1703606], in the case with knots.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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