Let $M$ be a regularly connected, finite, $n$-dimensional cellular polyhedron equipped with a metric $r$ which is compatible with the topology of $M$, and $\mu$ be a regular, non-atomic measure on $M$ which is zero on the set of non-regular points of $M$ and positive on any open subset of the set of regular points of $M$. $H(M, \mu)$ denotes the group of $\mu$-preserving homeomorphisms of $M$ with the topology induced by the metric $\rho(S, T) = \operatorname{Max}_{x \in M} r(Sx, Tx)$. Following their earlier paper [Uspehi Mat. Nauk 22 (1967), no. 5 (137), MR0219697], the authors define cyclic approximation by periodic transformations (a.p.t.) with speed $f(n)$ as follows: an automorphism $T$ of the Lebesgue space $(M, \mu)$ admits a cyclic a.p.t. with speed $f(n)$ if $0 \leq f(n) \searrow 0$ ($n \to \infty$) and there exist a sequence of measurable partitions $\xi_n$ of $M$ and of $\mu$-preserving transformations $T_n$ such that (A.1) $\xi_n \to \varepsilon$, the point partition ($n \to \infty$), i.e., for each measurable set $E$ there exists a sequence of $\xi_n$-sets $E_n$ so that $\mu(E_n \Delta E) \to 0$ ($n \to \infty$), (A.2) $T_n \xi_n = \xi_n$, (A.3) $\sum_{\xi_n} \mu(T \xi_n \Delta T_n \xi_n) < f(q_n)$, where $q_n$ is the cardinality of $\xi_n$, (A.4) $T_n$ cyclically permutes the elements of $\xi_n$.

Theorem A: automorphisms in $H(M, \mu)$ which admit a cyclic a.p.t. with some speed $f(n)$ form an everywhere dense $G_\delta$ in $H(M, \mu)$.

Theorem B: the set of automorphisms with continuous spectrum is an everywhere dense $G_\delta$ in $H(M, \mu)$. Corollary 1: the set of automorphisms with simple, singular and continuous spectrum is an everywhere dense $G_\delta$ in $H(M, \mu)$. Corollary 2: The ergodic homeomorphisms in $H(M, \mu)$ form an everywhere dense $G_\delta$ in $H(M, \mu)$.


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