Let $G$ be either a semisimple Lie group with noncompact factors of real rank at least 2, or an irreducible lattice in such a group. A group $G$ of this sort admits some notable actions of “algebraic type”. These are given by double cosets of the form $K\backslash H/\Lambda$, where $H$ is an algebraic group with $\Lambda$ a lattice and $K$ a compact subgroup. Assuming that $G$ (or the semisimple Lie group in which it is a lattice) is contained in $H$ and centralizes $K$, such double cosets admit a finite volume-preserving action of $G$. The main conjecture in the setup of Zimmer’s program is that every volume-preserving $G$-action is “essentially” a finite union of such double coset $G$-actions of algebraic type.

The authors provide a positive result which is free of some of the restrictive conditions usually considered in other articles. However, an important restriction of this work is the assumption that the actions considered are analytic.

For $n \geq 3$, denote with $\rho_0$ the natural $\text{SL}(n, \mathbb{Z})$-action on the torus $\mathbb{T}^n$. This induces an $\text{SL}(n, \mathbb{Z})$-action on the group $\pi_1(\mathbb{T}^n) = \mathbb{Z}^n$, which the authors call the standard homotopy data. Let $\Gamma \subset \text{SL}(n, \mathbb{Z})$ be a finite index subgroup and suppose that we are given a $\Gamma$-action on $\mathbb{T}^n$ which we will denote by $\rho$. The induced $\Gamma$-action on $\pi_1(\mathbb{T}^n) = \mathbb{Z}^n$ is called the homotopy data of $\rho$. Also, $\rho$ is said to have standard homotopy data if the homotopy data of $\rho$ is a restriction of the standard homotopy data.

The main result proves that if the $\Gamma$-action $\rho$ on $\mathbb{T}^n$ is analytic, has standard homotopy data and preserves an ergodic measure whose support is not contained in a ball, then $\rho$ and $\rho_0$ are analytically conjugate up to a finite index subgroup of $\Gamma$ and a conull subset of $\mathbb{T}^n$. Some additional properties of the conjugacy are also established.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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