
A sharp estimate for the smoothness of a conjugation that linearizes a nonlinear circle diffeomorphism. (Russian)


From the text (translated from the Russian): “We prove that any $C^{2+\alpha}$-smooth orientation-preserving circle diffeomorphism with a rotation number in the Diophantine class $D_\delta$, $0 < \delta < \alpha \leq 1$, is $C^{1+\alpha-\delta}$-smoothly conjugate to the corresponding linear rotation of the circle. This result gives, for the first time, a sharp estimate for the smoothness exponent of the linearizing change of coordinates. We also strengthen the Denjoy inequality for this class of diffeomorphisms.

An irrational number $\rho$ is said to belong to the Diophantine class $D_\delta$, $\delta \geq 0$, if there exists a $C > 0$ such that $|\rho - \frac{p}{q}| \geq Cq^{-2-\delta}$ for any rational number $\frac{p}{q}$.

Theorem 1. Let $T$ be a $C^{2+\alpha}$-smooth orientation-preserving circle diffeomorphism with rotation number $\rho \in D_\delta$, $0 < \delta < \alpha \leq 1$. Then $T$ is conjugate to the rotation of the circle through an angle $\rho$ by means of a $C^{1+\alpha-\delta}$-smooth change of coordinates.

This result was announced in [Y. G. Sina˘ı and K. M. Khanin, Uspekhi Mat. Nauk 44 (1989), no. 1(265), 57–82, 247; MR0997684], but its complete proof had not yet been published.

Our approach in proving this result is conceptually new. In fact, the proof is elementary in the sense that it uses no methods other than combinatorial considerations and elements of mathematical analysis. Despite this, our method leads to the strongest result. Nevertheless, the reader is expected to be familiar with the (itself elementary) Denjoy theory on the continuous linearization of circle diffeomorphisms of smoothness $C^2$ and higher.”

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