Smooth linearization of commuting circle diffeomorphisms. (English summary)


This important paper considers the problem of simultaneously conjugating a finite set of commuting orientation-preserving diffeomorphisms $f_1, \ldots, f_d$ of the circle to rotations and the smoothness of the conjugating homeomorphism $h$. The assumptions are in terms of arithmetic conditions on the rotation numbers of $f_1, \ldots, f_d$. Earlier results by Arnold and Moser also included the assumption that the $f_j$ lie in a small neighborhood of the corresponding rotations.

We denote the distance of the real number $x$ from the nearest integer by $\|x\|$ and the rotation $z \mapsto e^{2\pi i \theta} z$ of the unit circle by $R_{\theta}$.

The following fundamental result is proved. Suppose that $d \geq 2$ and $\theta_1, \ldots, \theta_d \in (0, 1)$ are such that there exist $\nu$ and $C > 0$ so that for each nonzero integer $k$, we have

(1) \[ \max\{\|k\theta_1\|, \ldots, \|k\theta_d\|\} \geq C|k|^{-\nu}. \]

Suppose that $f_1, \ldots, f_d$ are commuting ($f_p \circ f_j = f_j \circ f_p$ for all $p, j$) orientation-preserving infinitely many times differentiable diffeomorphisms of the circle such that $f_j$ has rotation number $\theta_j$. Then there exists an infinitely many times differentiable diffeomorphism $h$ of the circle conjugating all the $f_j$ simultaneously to rotations, that is, $h \circ f_j \circ h^{-1} = R_{\theta_j}$ for all $j$ with $1 \leq j \leq d$. If all $f_j$ are real analytic, then $h$ will also be real analytic.

In addition, if (1) is not satisfied, then there are commuting, infinitely many times differentiable, diffeomorphisms $f_1, \ldots, f_d$ with rotation numbers $\theta_1, \ldots, \theta_d$ that can be simultaneously conjugated to rotations by a homeomorphism $h$ but such that $h$ is not absolutely continuous.

Further, concerning the real analytic case, the authors point out that if the $\theta_j$ are given so that for some $a \in (0, 1)$ and for infinitely many $k \geq 1$, we have $\max\{\|k\theta_1\|, \ldots, \|k\theta_d\|\} \leq a^k$, then there exist commuting real analytic diffeomorphisms $f_1, \ldots, f_d$ with rotation numbers $\theta_1, \ldots, \theta_d$ that can be simultaneously conjugated to rotations by a homeomorphism $h$ but such that $h$ is not absolutely continuous. For a single real analytic diffeomorphism, the smoothness-of-conjugation problem in terms of the rotation number was settled by J.-C. Yoccoz [in Dynamical systems and small divisors (Cetraro, 1998), 125–173, Lecture Notes in Math., 1784, C.I.M.E., Florence, 2002; MR1924912].

For the proofs, the authors develop ways of estimating derivatives of diffeomorphisms.

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References

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12. J.-C. YOCCOZ, Analytic linearization of circle diffeomorphisms, in Dynamical Systems and Small Divisors (Cetraro, 1998), Lecture Notes in Math. 1784, Springer-Verlag, New York, 2002, pp. 125–173. (Received September 30, 2006) (Revised October 26, 2007) E-mail address: fayadb@math.univ-paris13.fr INSTITUT GALILEE, UNIVERSIT PARIS 13, AVENUE JEAN-BAPTISTE CLMENT, 93439 VIL- LETANEUSE, FRANCE PARIS 13, AVENUE JEAN-BAPTISTE CLMENT, 93439 VIL- LETANEUSE, FRANCE E-mail address: khanin@math.toronto.edu DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TORONTO, ROOM 6290, 40 ST. GEORGE STREET, TORONTO, ON M5S 2E4, CANADA

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