M. G. Khovanov’s combinatorial SL₂ link homology theory assigns to each link \( L \subset S^3 \) a bigraded abelian group, up to isomorphism [Duke Math. J. 101 (2000), no. 3, 359–426; MR1740682]. P. Seidel and I. Smith’s ‘symplectic Khovanov homology’ [Duke Math. J. 134 (2006), no. 3, 453–514; MR2254624] attaches to each link \( L \subset S^3 \) a singly graded abelian group, defined by applying the machinery of symplectic Floer cohomology to the monodromy of a symplectic fibration over the configuration space of \( 2n \) points in \( \mathbb{C} \). (The connection with links arises because loops in the base of this fibration represent braids.) Seidel and Smith conjectured that symplectic Khovanov homology is isomorphic to Khovanov homology with its bigrading collapsed.

With that conjecture in mind, the present paper develops additional structure in the Seidel-Smith theory, parallel to structure in Khovanov homology.

For each integer \( n \geq 0 \), Khovanov [Algebr. Geom. Topol. 2 (2002), 665–741; MR1928174] defined an associative algebra \( H^m \), with a distinguished collection of idempotents indexed by crossingless matchings in a half-plane of \( 2n \) points on a line. To each tangle \( T \) with \( 2m \) incoming and \( 2n \) outgoing strands he assigns a complex of graded \((H^m, H^n)\)-bimodules, up to quasi-isomorphism. These complexes satisfy a composition law under concatenation of tangles.

On the symplectic side, the present paper develops invariants of even tangles in a format similar but not identical to Khovanov’s. Two closely-related versions are given, one abstract, the other concrete. In the abstract version, one forms the extended Fukaya \( A_\infty \)-categories of the exact symplectic manifolds \( Y_n \) from which symplectic Khovanov homology is defined. The invariant is a functor between such extended categories, constructed by a mechanism due to K. Wehrheim and C. T. Woodward [Quantum Topol. 1 (2010), no. 2, 129–170; MR2657646]. The concrete version assigns to \( T \) a graded abelian group, constructed using quilted Floer cohomology [K. Wehrheim and C. T. Woodward, Geom. Topol. 14 (2010), no. 2, 833–902; MR2602853], analogous to the cohomology of Khovanov’s complex. Consideration of multiplicative structures on this sum of Floer cohomology groups is deferred to a future paper.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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