The authors “wish to determine the precision and stability of a (long) computation with respect to the round off errors.” They take as an example the problem of inverting high order matrices \( n \sim 100 \) by the elimination method. This is the most appropriate method for their aim, first because the inversion of such matrices plays an important role in the introduction of new procedures, second because it shows the problem of accumulation of round errors in “Reinkultur,” as it is an elementary one, having no difficulties of transcendentality, like differential equations. As for high-speed calculating machines, otherwise than for normal machines, a continuous inspection and supervision of the calculation is undesirable or impossible, the authors restrict the analysis to definite matrices since only for them can the question of precision be put on a rigorous basis. Then an arbitrary matrix \( M \) must first be transformed into a definite one \( M' M \).

The case of a definite matrix \( A \) is treated by an elaborate rigorous analysis. Its essential tricks are as follows. First, \( A \) can be decomposed into \( A = B'DB \), where \( B \) is triangular and \( D \) is diagonal. Second, even when the resulting approximations of \( B \) and \( D \) to their “true” values are poor, the decomposition itself is still good within an error of \( 0.42n^2 \times 10^{-s} \) in terms of the bound of the matrix \( (A - B'DB) \), where \( s \) denotes the fixed number of digits used in the calculation. This estimate is only possible for a definite \( A \). Finally the authors obtain an approximant \( 2^q W \) to \( A^{-1} \) and show that the approximation is good within an error at most \( 14(\lambda/\mu)n^210^{-s} \), where \( \lambda \) and \( \mu \) are the largest and smallest proper values of \( A \). The algorithm itself is such that either the approximate inverse is found or it is discovered that \( \mu < n^210^{-s+1} \), i.e., that \( A \) is approximately singular. For a general orientation about the conditions that prevail for \( n \)th order matrices, the authors use statistical estimates of \( \lambda, \mu \) and put \( \lambda \leq 100n, \mu \geq 1/(100n) \). Thus an approximate inverse will usually be found if \( n < 0.15 \times 10^{n/4} \). By this result the usual opinions as to the precision required when inverting matrices of high order are refuted. 

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