Causes of stretching of Birkhoff sums and mixing in flows on surfaces. (English summary)


Let $Tx = x + \alpha$ be an irrational rotation on the circle $T = [0,1)$ and $f: T \to \mathbb{R}$ a positive function of integral 1. The mixing property of the corresponding special flows is studied; in fact, a stronger phenomenon called the stretching of Birkhoff sums, which are $f^{(r)}(x) = \sum_{k=0}^{r-1} f(T^k x)$, is examined. This phenomenon causes, as time passes, a small rectangle to become strongly stretched and almost uniformly distributed along trajectories of the special flow, whence the mixing property can be deduced. The author of the paper under review presents the main ideas of proving the stretching of Birkhoff sums in three cases that appeared in earlier papers:

1. $f$ is a particular (Hölder) continuous function [A. V. Kochergin, Mat. Sb. 193 (2002), no. 3, 51–78; MR1913598];
2. $f$ has finitely many logarithmic singularities and is asymmetric [Y. G. Sinaï and K. M. Khanin, Funktsional. Anal. i Prilozhen. 26 (1992), no. 3, 1–21; MR1189019; A. V. Kochergin, Mat. Sb. 194 (2003), no. 8, 83–112; MR2034533];

In the first two cases some Diophantine conditions are imposed on $\alpha$.

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