In this paper the author studies a problem of model reduction, known in many physical systems, for a diffusively-perturbed pseudoperiodic flow on the 2-dimensional torus.

In [Funktsional. Anal. i Prilozhen. 25 (1991), no. 2, 1–12, 96; MR1142204] V. I. Arnold identified the general structure of such flows. He showed that there is a partition of the torus into an ergodic region and a collection of traps. Inside each of the traps, the flow can be described via a local Hamiltonian.

The author considers the following assumptions:

(H1) Let \( H \in C^\infty(\mathbb{R}^2) \) and \( \omega = (\omega_1, \omega_2) \in \mathbb{R}^2 \) be such that \( H \) is Morse, \( \omega_1 \) and \( \omega_2 \) are incommensurable, i.e. \( \langle \omega, K \rangle_{\mathbb{R}^2} \neq 0 \) for all \( K \in \mathbb{Z}^2 \setminus \{(0,0)\} \subset \mathbb{R}^2 \), \( H(x + K) = H(x) + \langle \omega, K \rangle_{\mathbb{R}^2} \) for all \( x \in \mathbb{R}^2 \) and \( K \subset \mathbb{Z}^2 \subset \mathbb{R}^2 \), and the vector field \((\nabla \phi)(x) := \left( \frac{\partial H}{\partial x_2} \frac{\partial \phi}{\partial x_1} - \frac{\partial H}{\partial x_1} \frac{\partial \phi}{\partial x_2} \right)(x)\) for \( \phi \in C^\infty(\mathbb{R}^2) \) and \( x = (x_1, x_2) \in \mathbb{R}^2 \).

(H2) Consider \((\mathcal{L}f)(x) := \frac{1}{2} \sum_{i,j \in \{1,2\}} a^{(2)}_{ij}(x) \frac{\partial^2 f}{\partial x_i \partial x_j}(x) + \sum_{i \in \{1,2\}} a^{(1)}_i(x) \frac{\partial f}{\partial x_i}(x)\) and \( \mathbb{T} := \mathbb{R}^2/\mathbb{Z}^2 \) to be the two-dimensional torus and the Markov process on \( \mathbb{T} \) whose generator is \( \mathcal{L} := \varepsilon^{-2} \mathcal{L} + \mathcal{L} \).

Equipped with these assumptions, the author is interested in how small diffusive perturbations cause transitions between the traps and the ergodic class. The interesting part is the effect of bifurcations, which create different strata, thus the focus of this paper is the effect of the ergodic class.

The stochastic averaging suggests that he looks for closed dynamics of the action variable. Under an appropriate change of time, the author identifies a reduced model as the strength of the random perturbation tends to zero along a certain subsequence. He uses arguments of Freidlin and Wentzell to identify a limiting graph-valued process in the Hamiltonian region. Thus he reduces the ergodic region to a single point, which is sticky and the identification of the glueing conditions follows from a perturbed test-function analysis in the ergodic region.

The author studies, in detail, some classical results of tightness, partial differential equations, Markov processes; perturbations of Hamiltonian systems, and stratified Morse theory and functional analysis in connection with stochastic averaging.

The material is unavoidably technical, and the author has gone to great lengths to motivate the material with some discussions about strategy.

This memoir will be of lasting value in the functional analysis and random perturbations of pseudoperiodic flows. The approach and result on the important problem in this work will appeal to researchers and graduate students in random perturbations, Hamiltonian systems, and mathematical physics.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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