In the “autonomous case”, i.e., when there exists $\varepsilon > 0$ such that $\Delta \varphi + \lambda \varphi = V(x)g(\varphi)$. 

The authors study the existence and orbital stability of standing waves for the nonlinear Schrödinger equation

$$(A) \quad iu_t + \Delta u = V(x)g(u),$$

where $(t, x) \in \mathbb{R} \times \mathbb{R}^N$, $N \geq 3$, $u \in H^1(\mathbb{R}^N, \mathbb{C})$, $V$ is a real-valued potential and $g$ is a nonlinearity with $g(e^{i\theta} s) = e^{i\theta} g(s)$, $s \in \mathbb{R}$. A solution of the form $u(t, x) = e^{i\lambda t} \varphi(x)$ is called a standing wave. For solutions of this type with $\varphi \in H^1(\mathbb{R}^N, \mathbb{R})$ the above Schrödinger equation $(A)$ is equivalent to the equation

$$(B) \quad -\Delta \varphi + \lambda \varphi = V(x)g(\varphi).$$

In the “autonomous case”, i.e., when $V(x) = \text{constant}$, we refer to the work of H. Berestycki and P.-L. Lions [Arch. Rational Mech. Anal. 82 (1983), no. 4, 313–345; MR0695535]. The main hypotheses are $0 < b < 2$, $1 < p < 1 + \frac{4-2b}{N}$, (H$_1$) there exists $\gamma > 2N/(N+2)-(N-2)p$ such that $V \in L^\gamma_{loc}(\mathbb{R}^N)$; (H$_2$) $\lim_{|x| \to \infty} V(x)|x|^b = 1$; (H$_3$) there exists $\varepsilon > 0$ such that $g: [0, \varepsilon] \to \mathbb{R}$ is continuous; (H$_4$) $\lim_{s \to 0^+} \frac{g(s)}{s} = 1$; (H$_5$) $g \in C^1(\mathbb{R}, \mathbb{R})$; (H$_6$) there exist $C > 0$ and $\alpha \in [0, \frac{4}{N-2})$ such that $\lim_{s \to \infty} \frac{|g'(s)|}{s^\alpha} \leq C$; (H$_7$) $\lim_{s \to 0^+} \frac{g'(s)}{ps^{\alpha-1}} = 1$, where $p$ is given in (H$_4$). Finally, two main results are gained: (A) Under hypotheses (H$_1$)–(H$_4$) there exists $\lambda_0 > 0$ such that for all $\lambda \in (0, \lambda_0]$ problem (B) has a nontrivial solution $\varphi_\lambda \geq 0$ with $\|\varphi_\lambda\|_{H^1(\mathbb{R}^N)} \to 0$ and $\|\varphi_\lambda\|_{L^\infty(\mathbb{R}^N)} \to 0$, as $\lambda \to 0$. (B) Under hypotheses (H$_1$)–(H$_7$) there exists $\lambda_1 > 0$ such that for all $\lambda \in (0, \lambda_1]$ the travelling wave $e^{i\lambda t} \varphi_\lambda(x)$ is stable in $u \in H^1(\mathbb{R}^N, \mathbb{C})$. These results may be considered as a generalization of some earlier works of the first author [see L. Jeanjean and K. Tanaka, Indiana Univ. Math. J. 54 (2005), no. 2, 443–464; MR2136816; and references therein].

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References


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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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